ADAPTING THE ABACUS
FOR THE USE OF THE BLIND

by

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TABLE OF CONTENTS

LIST OF FIGURES AND ILLUSTRATIONS

CHAPTER I

THE TEACHING OF MATHEMATICS TO THE BLIND

Since the time of the first organized attempts to educate the blind the teaching of mathematics has proved to be one of the most formidable challenges faced by the workers in this field. In 1944 Hayes (9) pointed out the relatively poor level of achievement displayed by the students in thirteen schools for the blind. He stated that arithmetic had nearly always ranked poorest among the ten parts of the Stanford Achievement Test Series which had been administered. The serious implications of these findings were described by Hayes (9:99) in the following manner:

> "Various schools which have given tests year after year have shown themselves able to overcome weaknesses in many subjects through better instruction or motivation, but in these same schools the curve continues to drop suddenly downward for the scores in arithmetic computation."

Why do blind students as a group rank lower in arithmetic than in other subjects when compared with the achievement norms of the sighted? Are the mathematical skills so dependent upon visual aids and impressions that lack of sight results in a lack of proficiency in numerical calculations? At present the answers to these questions are shrouded in controversy, and there-

fore, it is hoped that this study will aid in clarifying them to the advantage of those who are most concerned, the blind themselves.

In considering the achievement of any blind group the range of individual differences will be as great as that found in sighted groups. Achievement in arithmetic is definitely not an exception to this rule. Some of the implications of this fact were recognized by Hayes (9:100) when he observed that:

> "Even in the schools which do poorly in the tests we find a small group of pupils who test well above their grade standards, and the fact that some blind men have become successful or even famous mathematicians, proves that blindness is not an insuperable obstacle in this field."

The effects of individual differences in the classroom situation are described by McCutchan (14:41):

> "In no other class do I feel that there is as great a need for the adjustment of bright and dull students as in Arithmetic. Dull children need considerably more explanation and many more concrete illustrations. They require more repetition or drill work in order to fix a certain process in their minds. Bright children object to very much drill material."

Thus, it may be concluded that the teacher of mathematics must cope, not only with the problems associated with blindness, but also with those arising from the existence of significant individual differences in the learning ability of blind students. Any attempt

to improve the achievement of the blind as a group should not only meet the needs of the slow learner, but should challenge the abilities of the rapid learner.

Various methods found useful in building up a basic understanding of number concepts have been presented by Fuchs (7). He suggests that the blind child be guided into the abstractions of numerical calculation by means of concrete objects which form a part of the pupil's everyday surroundings. Such objects as coins, sticks, and nuts may be organized as games and drills which will provide valuable foundations for quantitative thinking. McCutchan (14:41) supports this contention by stating that:

> "various number games serve as an excellent means of simplifying otherwise difficult number concepts."

The results of these concrete experiences must be utilized in the expression of number as a symbol. Symbolic mathematical notations can be made by the blind in the form of raised dots known as braille.

The development of the braille system has provided the most satisfactory means of tactual reading and writing now available to the blind. This system is based upon various combinations of a group of six raised dots. (Fig. 1) Reading and writing braille is a much slower process than that for ink print. The braille slate and

a b c d e f g h i j k l m

n o p q r s t u v w x y z

Number Sign

1 2 3 4 5 6 7 8 9

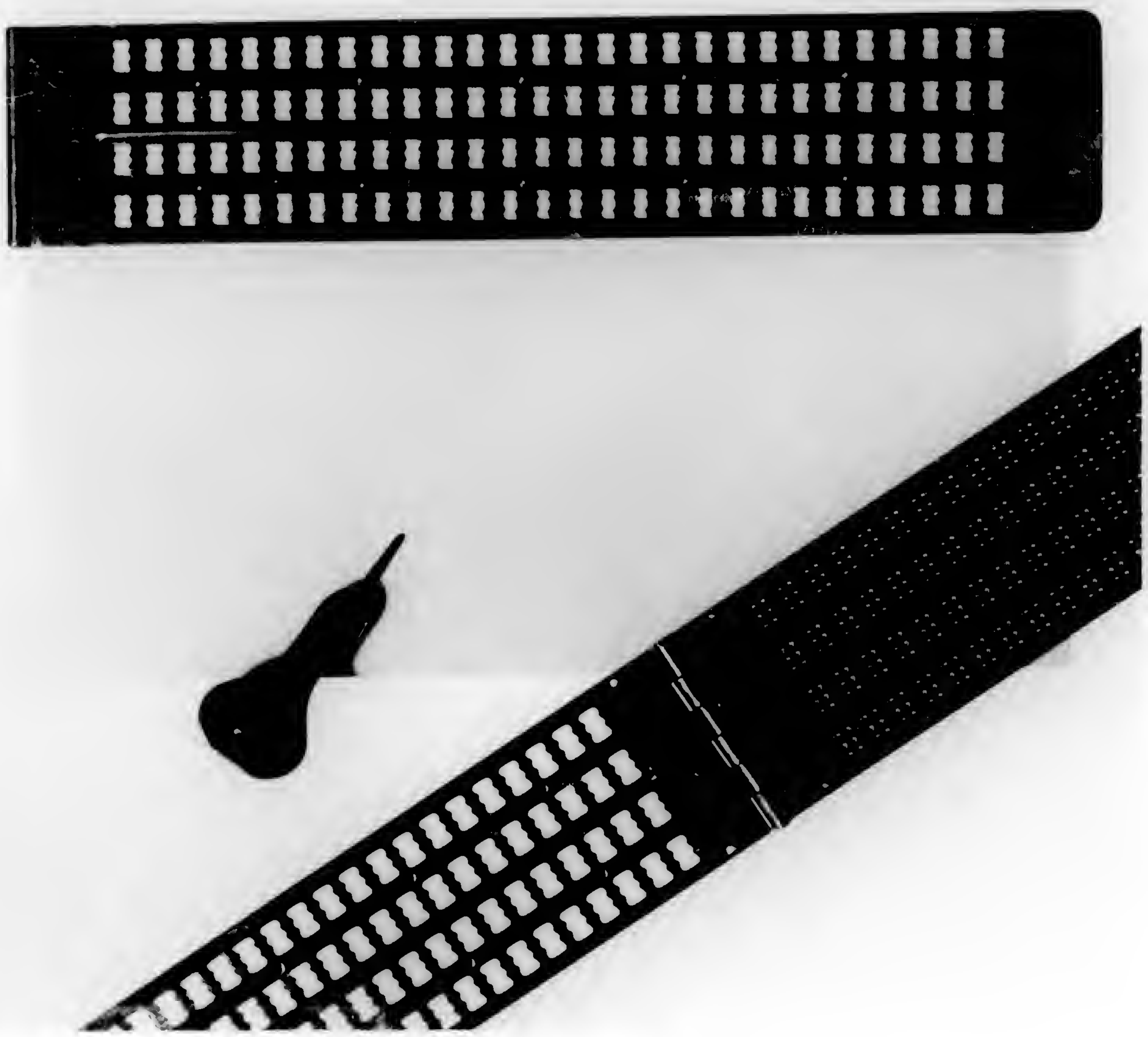
0

Fig. 1 - Braille Letters and Numbers As They Are Read

stylus (Illus. A) have been developed for the writing of braille letters and numbers. The stylus is designed to press out the raised dots and the braille slate to guide and shape their formation. All braille symbols must be written backwards on the opposite side of the paper from which they are read. (Fig. 2) This reversed process of writing is a mechanical necessity when using a braille slate and stylus.

The Arabic numberals are represented by the first ten letters of the braille alphabet preceded by four dots known as the number sign. (Fig. 1) When computing with braille numbers the blind mathematician encounters certain disadvantages which are not experienced by persons using pencil and paper. For instance, braille numbers cannot be written with a slate and stylus so that they may be read as they are written. The paper must be removed from the slate and turned over before the numbers which have been written can be read. The most commonly used methods of computation for pencil and paper are practically impossible to reproduce with a braille slate and stylus. These mechanical difficulties have forced teachers of mathematics to seek methods and devices which would meet the needs of their blind pupils in a more efficient manner.

The development of mental arithmetic as the ideal method for use with the blind has been favored by a



Illus. A - Braille Slate with Stylus

m l k j i h g f e d c b a

z y x w v u t s r q p o n

Number Sign

9 8 7 6 5 4 3 2 1

0

Fig. 2 - Braille Letters and Numbers As They Are Written

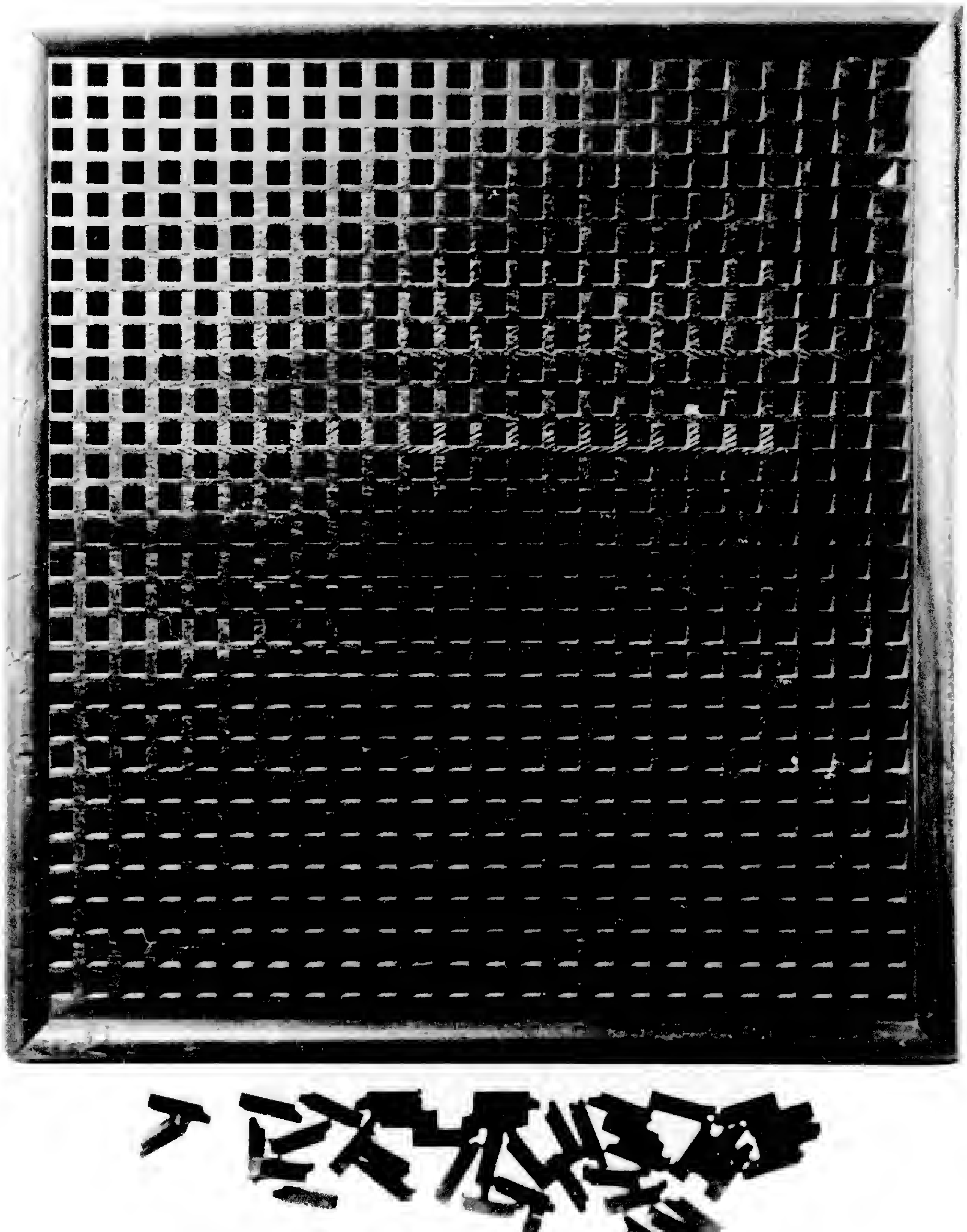
large group of instructors. However, there is much disagreement among them regarding the mental aspects of this method. Should the mind be required to carry the total burden envolved in computation? If not, what aids should be provided and how should they be used? Such questions as these have caused so much difference of opinion that mental arithmetic has developed in a very disorganized manner.

The mental arithmetic movement has suffered most from the mental limitations of the human mind. Complex problems often tax the memory more than the computing skills of blind mathematicians. The number of steps required in the solving process are often so numerous that the mind becomes overburdened with figures that are only incidental to the final answer. This additional strain handicaps accuracy as well as speed in computation. Consequently, the braille slate has been used whenever possible to record any figures which could not easily be remembered. This aid is undoubtedly better than nothing, but as pointed out previously, it possesses several disadvantages which slow down the process of computation.

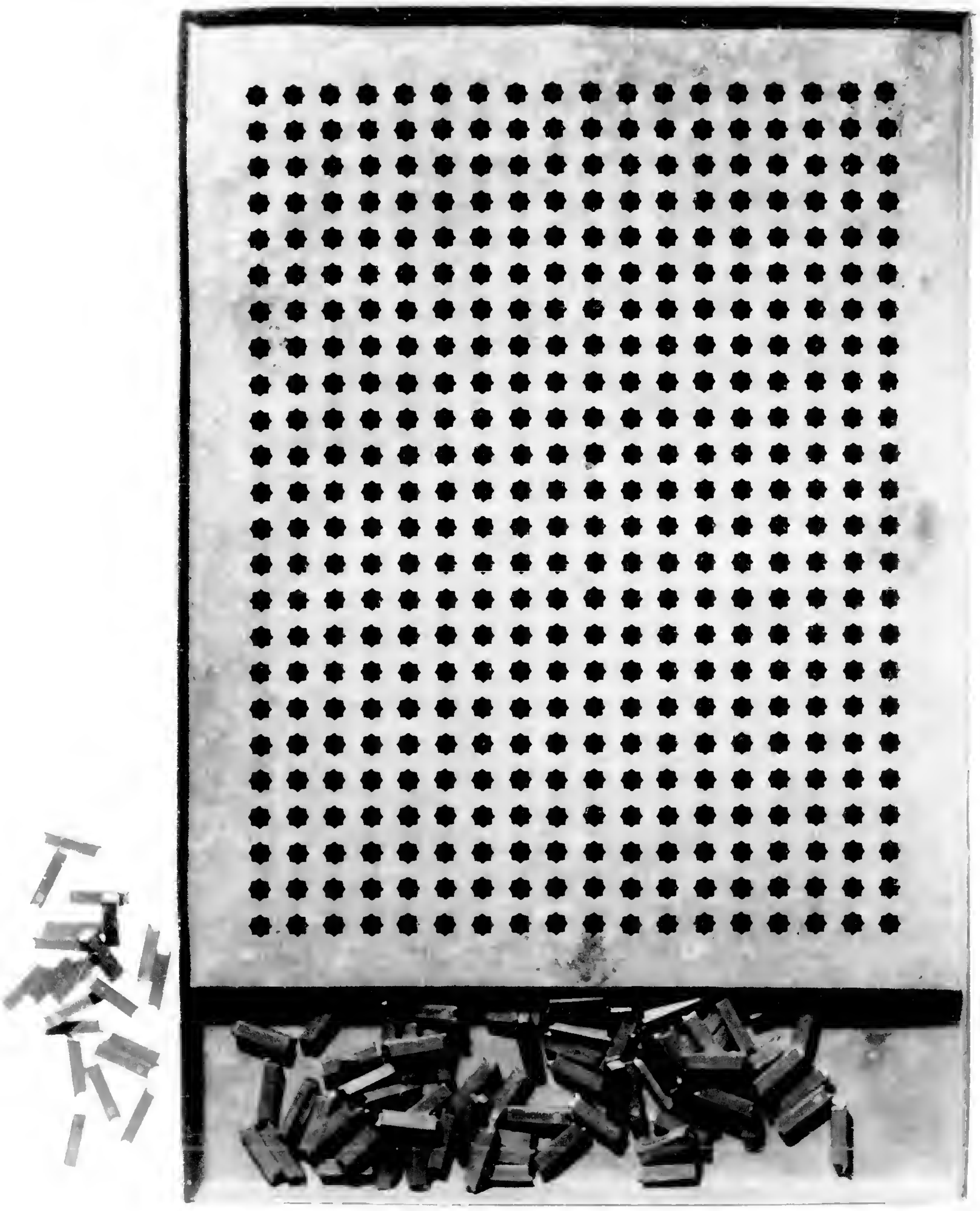
Considerable experimentation has been directed toward the objective of teaching mathematics to the blind by the same methods generally used to teach

sighted students. In this method the positions in which the numbers are placed have as much importance as do their actual quantitative values. Therefore, a means of tactual notation other than braille was sought by which numbers could be placed as with pencil and paper. This need brought about the development of various types of arithmetic slates. Hundreds of holes were arranged in a frame in a pattern similar to the squares on a chessboard. Numbers are represented on the frame by metal type which fit into the proper holes.

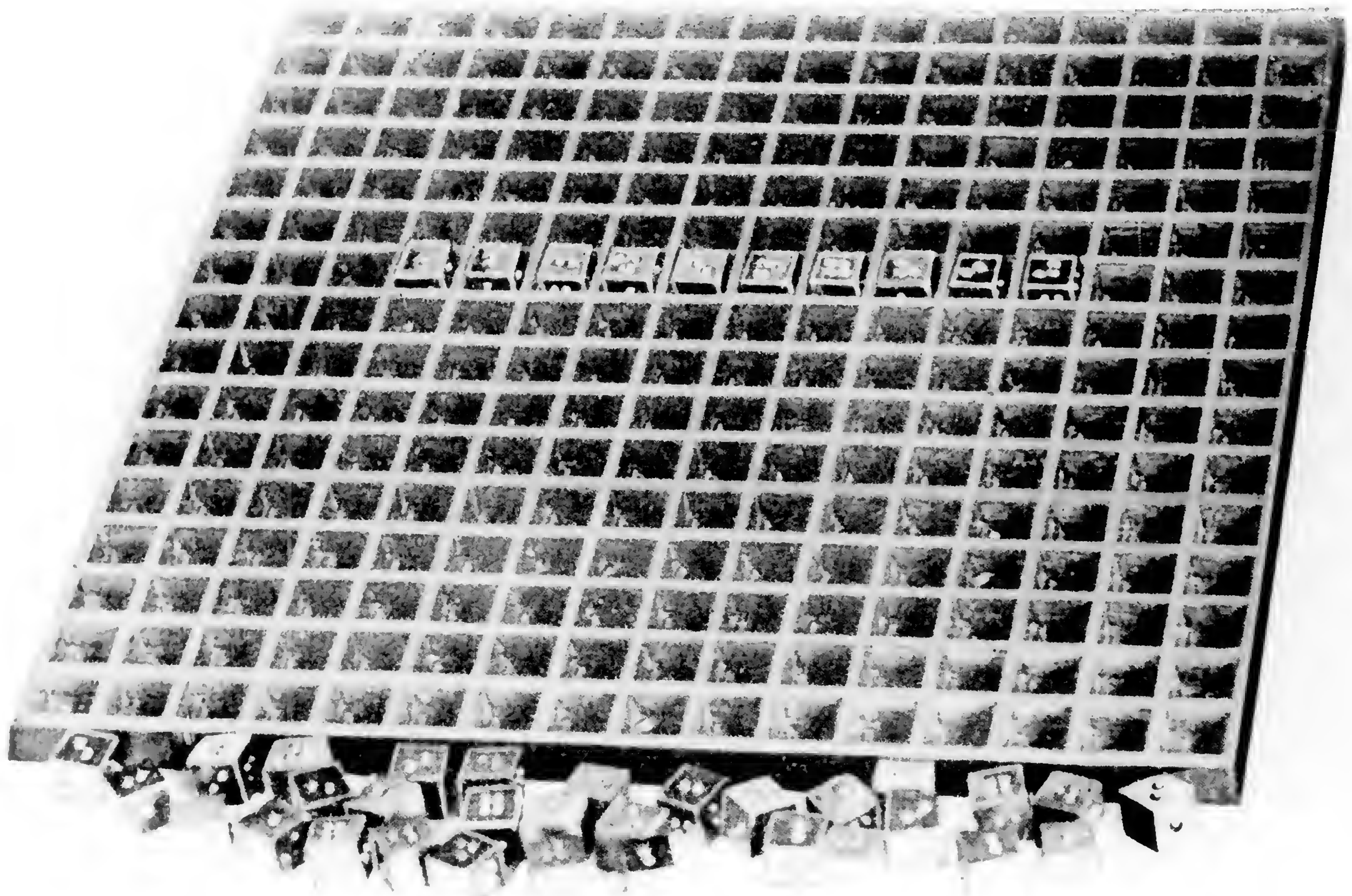
The two most common forms of type for arithmetic slates are: (a) type with raised lines that represent numbers depending upon the position or way that they are put into the holes; (b) type with braille numbers. The first form has the shape of a parallelepiped with four rectangular sides and two square ends. The holes into which this type is placed range from square (Illus. B) to octagonal (Illus. C) in shape. As the type is turned in different positions in the multiple-sided holes the raised lines on the ends change position. Thus, there is created a numerical code system which must be memorized by the blind student before this type of arithmetic slate can be operated. The second form of type is cubic in shape. (Illus. D) The cubes are about one-half inch square and fit into square holes. Braille



Illus. B - T.V.L. Aritmmetic Slate



Illus. C - Taylor Arithmetic Slate



Illus. D - Cubarithme Slate

numbers are placed upon the sides of each cube. This arrangement eliminates the necessity of learning a special number code in order to operate the slate.

There is much controversy over the question of superiority among those prefering different forms of arithmetic slates. Merry (16) (15) (18) reports the results of some experimental comparisons that have been made in an attempt to settle this question. Other comparisons such as those made by Merry must be carried out, if conclusive findings are to be obtained. Some teachers feel that any type slate is superior to a braille slate. Others prefer a braille slate to any form of type slate.

A great many blind students have used arithmetic slates while attending school, but few continue using them after leaving school. This situation may be due to the characteristics of arithmetic slates rather than to the students who use them, but the problem of a general mathematical inadequacy of the blind in post school society still exists. Most teachers seem to agree on what the general objectives should be for mathematics courses, but differ radically in regard to the methods which will best accomplish these objectives. For instance, Schoonmaker (25) reported to the American Association of Instructors for the Blind that "in teach-

ing arithmetic to the blind the methods used and the results desired should be identical with those for the seeing child." She also stated that when type slates were used multiplication and division were done in the same way as with pencil and paper.

The most common criticisms which are made concerning type slates are the following: (a) they are so large and cumbersome that they cannot be carried about and used with convenience; (b) the loose type which fit into the frame are easily lost and misplaced; (c) in some cases a separate number code must be learned. This increases the possibility of error, as well as the length of time required in learning to use the slate; (d) students often become so dependent upon the aid of a type slate that they are mathematically helpless without one. In most instances this situation results from poor teaching procedure.

A survey of the literature concerning the teaching of mathematics to the blind, then, discloses one very persistent and well-established trend. It is evident that regardless of what methods of computation are employed, the blind mathematician often desires the assistance of a convenient recording device. Braille and arithmetic slates are most frequently used to meet this need, but both are found lacking in vari-

ous respects. Would it be possible to develop a mechanical aid that could be adapted to all of the various methods of computation now taught to the blind? Such an aid, if universally adopted, should pave the way to a more general agreement upon teaching procedures. Agreement in methodology should result in better teaching, better achievement, and better braille textbooks.

The purpose of this study is to discover what possibilities the abacus possesses as a mechanical aid in computation for the blind. For many centuries this instrument has been used and perfected by men of many races. Even today it holds an important position in competition with modern adding and calculating machines. Its operation is quite simple, very rapid, and largely tactual. Only slight structural modifications should be necessary to equip it for the service of the blind.

CHAPTER II

THE HISTORIC DEVELOPMENT OF THE ABACUS

An important part of the hostory of mathematics is concerned with the development of mechanical aids for numerical notation and calculation. The first aids of this type were small objects such as shells, pebbles, and sticks. These were used loosely as representative units of quantity, and in groups as collections of units. Knotted cords and notched sticks also have been popular recording devices. In this way man began to utilize the counting tools of nature in coping with his mathematical needs. He soon discovered that the most convenient of these counting tools were the fingers of his hands. Finger counting and finger notation are highly developed arts which still play an active role in modern business centers, as well as in the more primitive societies.

The ten fingers possessed by a pair of human hands may well have been the stimulus for the centuries of mathematical progress which resulted in the modern abacus and the decimal system. The powers of ten are presented mechanically by the abacus and symbolically by the decimal system. Arthur (2) has illustrated what he believes to be the evolutionary process which began with the hands of man and ended with the abacus which is now used in

China. (Fig. 3) He not only connects hand symbols with the abacus, but also presents the latter as the parent of modern calculating machines.

The time and place of origin for the abacus is not definitely known; however, most writers on this subject tend to agree in most respects with the views of Baxandall (5:7). He believes the abacus to be an invention of the Semitic races, which was adopted in very early times by the people of India. From India its use spread westward into the countries of Europe, and eastward into China and Japan.

The earliest form of abacus is described by Knott (12) as a board covered with fine sand or dust. This surface was ruled into columns upon which numbers could be marked by symbolic strokes or objects. As time passed many changes were made in the structural design of this counting board. Waxed surfaces replaced the sand and dust. Counting boards with permanently ruled or grooved columns were found superior to the waxed boards. Barnard (4) gives a detailed account of these boards and the casting counters which were used with them.

The modern forms of the abacus consist of a frame in which are mounted a variable number of parallel rods or wires. The beads or counters are mounted on these rods and can be easily moved along them to the positions desired. Each rod with its sliding counters represents

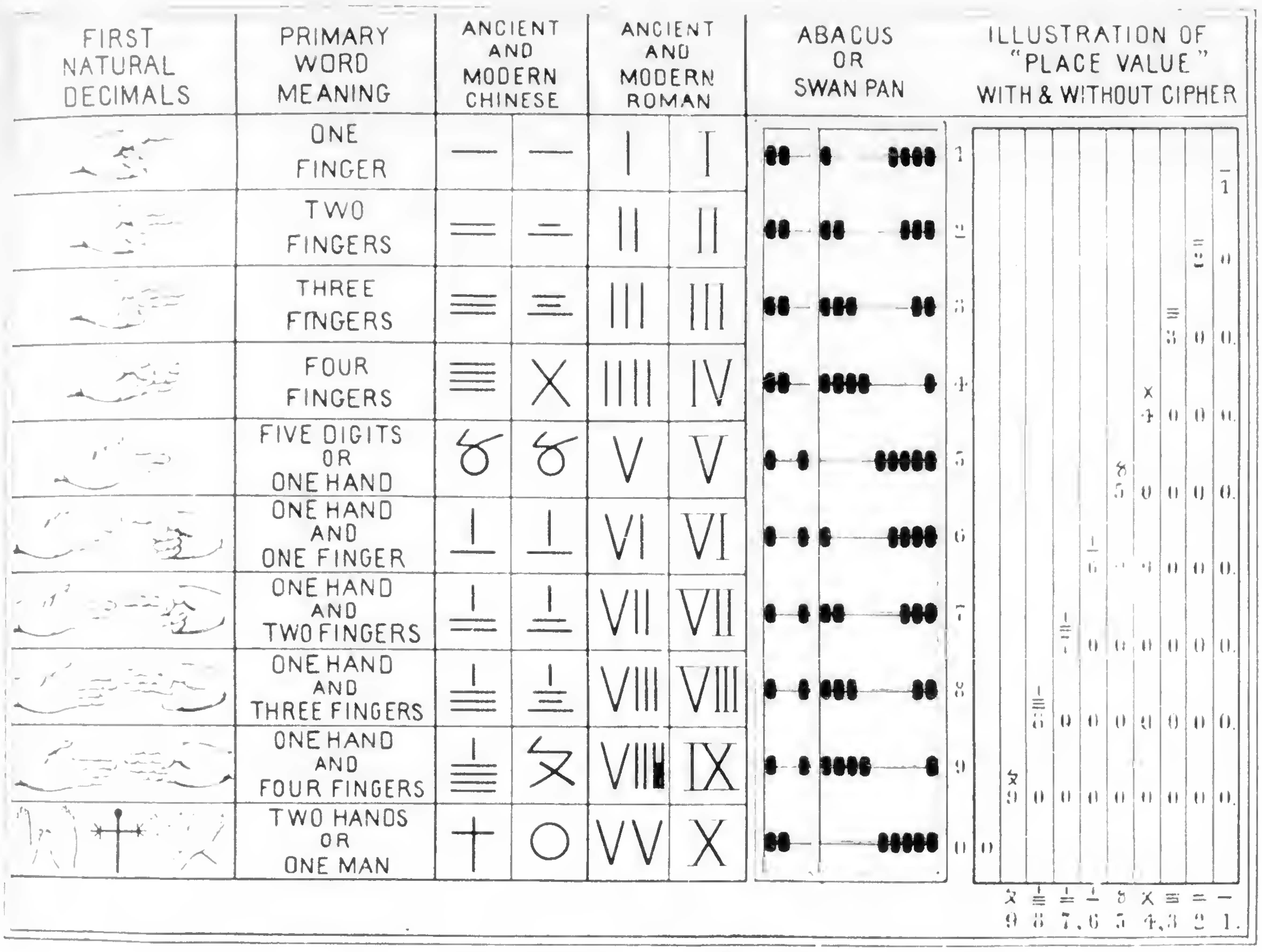


FIG. 3—SOME STEPS OF CENTURIES IN DECIMAL EVOLUTION.

one place in the decimal system. The greater the number of rods in a frame, the greater will be its number of possible decimal places. Thus, the symbolic abstractions of the decimal system are mechanically reproduced in a manner which is not only simple in structure but also numerically concrete.

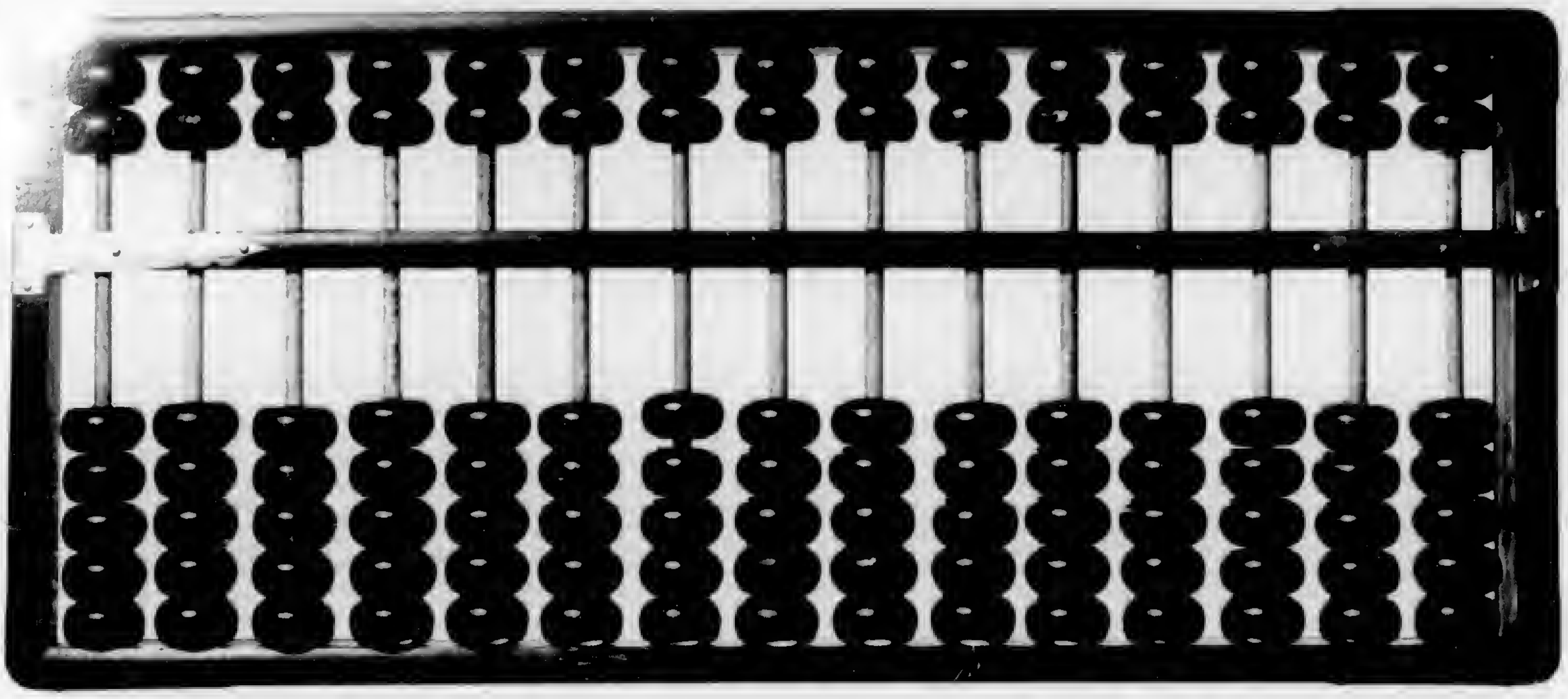
The modern abacus is still being used by a large percentage of the world's population. From its stronghold on the continent of Asia the abacus has been introduced into the cultures of many other lands. In China it is known as swan-pan, in Japan as soroban, in India as Suinbon, and in Russia as tschotz. Each of these differ somewhat in structural design and method of operation, but fundamentally they are the same instrument.

In many elementary schools a simple bead frame is used as an aid in teaching children to count. (Illus. G) This device closely resembles the Russian form of abacus. Upon each parallel wire of the frame are mounted ten sliding beads. The frame may be placed so that the wires are either in a vertical or a horizontal position. If the position were vertical, the wire farthest to the right would represent ones; the next tens; and then hundreds, etc. If their position were horizontal, the wire at the bottom of the frame would represent the ones' or units' place. The value of each bead is determined by-

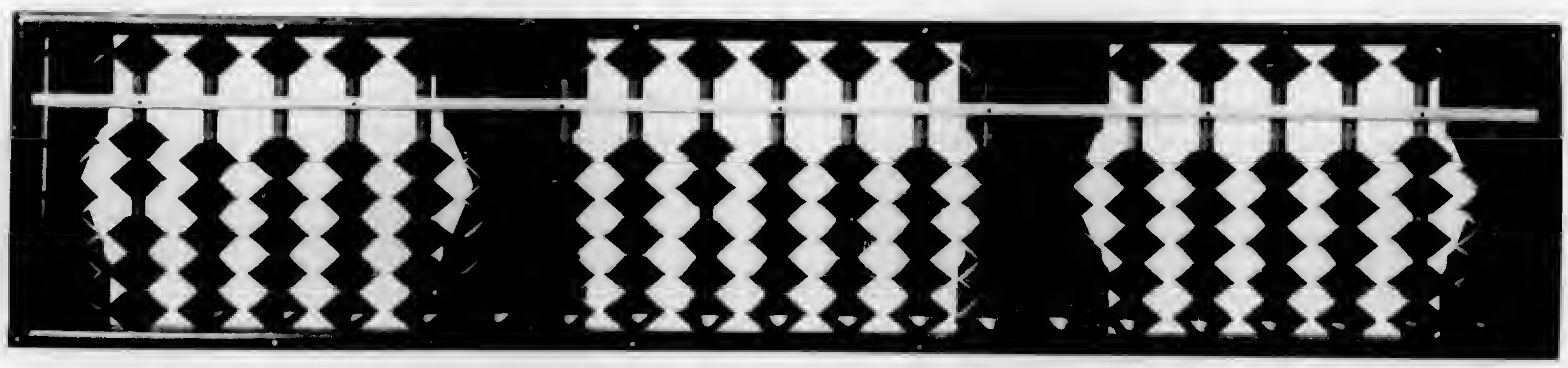
the relative position of its wire in the frame.

Spitzer (28) has described the construction and use of a small bead frame for teaching purposes at the State University of Iowa Elementary School. It possessed a numerical capacity of three decimal places. The counters were wooden beads from the ten-cent store. These beads were strung on pieces of coat hanger wire. This abacus was used to demonstrate certain outstanding features of modern arithmetic. Among these were the idea of place value, the collection idea of number, the function of zero, and the true character of borrowing and carrying. Spitzer considered the abacus to be an important teaching aid that should be used as a part of the instructional program in arithmetic.

The modern Chinese abacus is a more efficient computing device than the bead frame described by Spitzer. Instead of ten beads on each parallel wire, the _swan-pan_ (Illus. E) has but seven. These seven counters are divided into two groups by a horizontal strip which cuts the frame into two unequal compartments. Arthur (2) explains that the wider compartment contains five counters which represent the digits or fingers of a hand. The two beads in the narrow compartment symbolize a pair of hands. Each hand-bead has five times the value of a digit-bead. Beads are counted only



Illus. E - Chinese swan-pan



Illus. F - Japanese soroban

when they have been moved against the horizontal strip.

The swan-pan replaced the bamboo tallies or rods of the ancient Chinese during the fourteenth century. About three centuries later it was introduced into Japan and called the soroban. (Illus. F) Since that time the Japanese have made several structural changes in the design of the swan-pan. They bevelled the beads and placed them more closely together. Also one of the two upper beads was eliminated. There is some evidence which indicates that recently the soroban has lost one of its five digit beads. (1:35) This reduces the number of beads on each vertical rod to one above and four below the horizontal crosspiece.

At the present time there is considerable controversy over the true mathematical worth of the structural changes which have been made in the abacus by the Japanese. Chinese mathematicians have pointed out that changing the shape of the beads and decreasing the space between them interferes with speed and accuracy in computation. Arthur (2) believes that the elimination of one of the two beads in the upper compartment is a step in advance of the Chinese. He finds no use for the second counter in a strictly decimal operation. The same observation is made concerning the lowest row of digit beads. However, a study of Chinese works on the subject will disclose the fact that many of their cal-

culations are not decimal in nature. Therefore, it would seem that the structural design of the abacus is determined by the functions which it is expected to perform.

In the following chapters the mathematical needs of the blind shall determine what structural and methodological adaptations are necessary for the abacus as it is used today. Three different types will be considered. They are the <u>swan-pan</u>, the <u>soroban</u>, and the bead frame used in elementary schools.

CHAPTER III

STRUCTURAL MODIFICATIONS FOR USE WITH THE BLIND

The structural adaptation of the abacus for the use of the blind is primarily a problem of frictional adjustment. The beads or counters of an abacus are usually mounted very loosely upon their parallel rods. Freely sliding beads are designed to permit maximum speed in operation, but their manipulation must be extremely precise. Errors in computation result whenever counters are accidentally knocked out of position and not readjusted. Therefore, special methods of fingering the beads have been formulated to assist the unskilled beginner. The importance of correct fingering is emphasized by Kwa Tak Ming (21:7). In describing the operation of the <u>swan-pan</u> Yoshino (30:6) also devotes space to a discussion of this subject in his book on the <u>soroban</u>.

If the counters of the modern abacus are to be manipulated efficiently by blind operators a means must be found by which the frictional resistance against them may be increased. Without the aid of sight, generally accepted finger techniques are impractical guides when employed with beads that are easily displaced. The blind operator must depend en-

tirely upon tactual processes in developing his skill. Thus, the counters must be stabilized sufficiently to resist the dislocating effects of direct tactual contact.

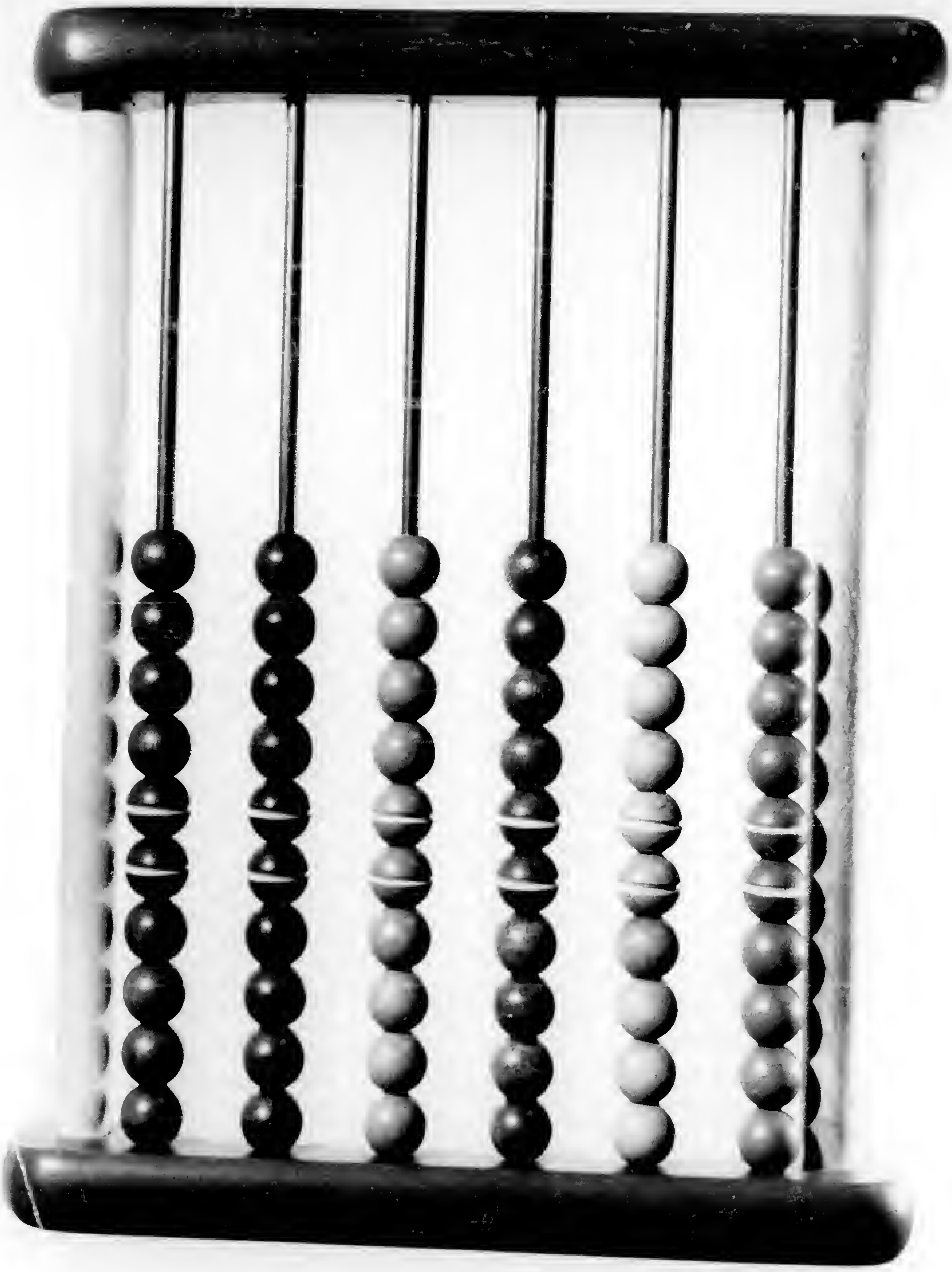
There are a number of possible ways by which the problem of stabilizing the sliding counters could be solved. All of these various methods may be listed under three headings: (a) creation of external friction between the beads and some other substance; (b) modification of the rods upon which the counters slide; (c) modification of the structural features of the beads.

Additional friction may be applied externally to the beads without the need for significant changes in the structural design of the abacus. This external friction is created by placing a resistant material such as sponge rubber against the under side of the instrument. There should be sufficient pressure against the beads to hold them in position when touched. If an excessive amount of pressure is applied to the beads they can be moved only with much effort. The resiliency of sponge rubber permits a bead to be moved along its rod without decreasing the pressure which holds the other beads in place. This method is most satisfactory for the larger sized

abacus such as the fifteen rod swan-pan. Its greatest disadvantage appears to be the increase in bulk of the original device.

Another means by which the beads of an abacus may be stabilized exists in certain structural modifications of the rods upon which the beads slide. When Oriental mathematicians demonstrate the operation of an abacus they often find it necessary to place it in a vertical position. In this position the counters normally fall to the bottom of the frame. Therefore, an adjustment was needed to hold the raised beads in position. Strips of plant fiber were often wound about the rods for this purpose. Plant stems with thick bristles sometimes were used to replace the smooth wooden rods of the abacus.

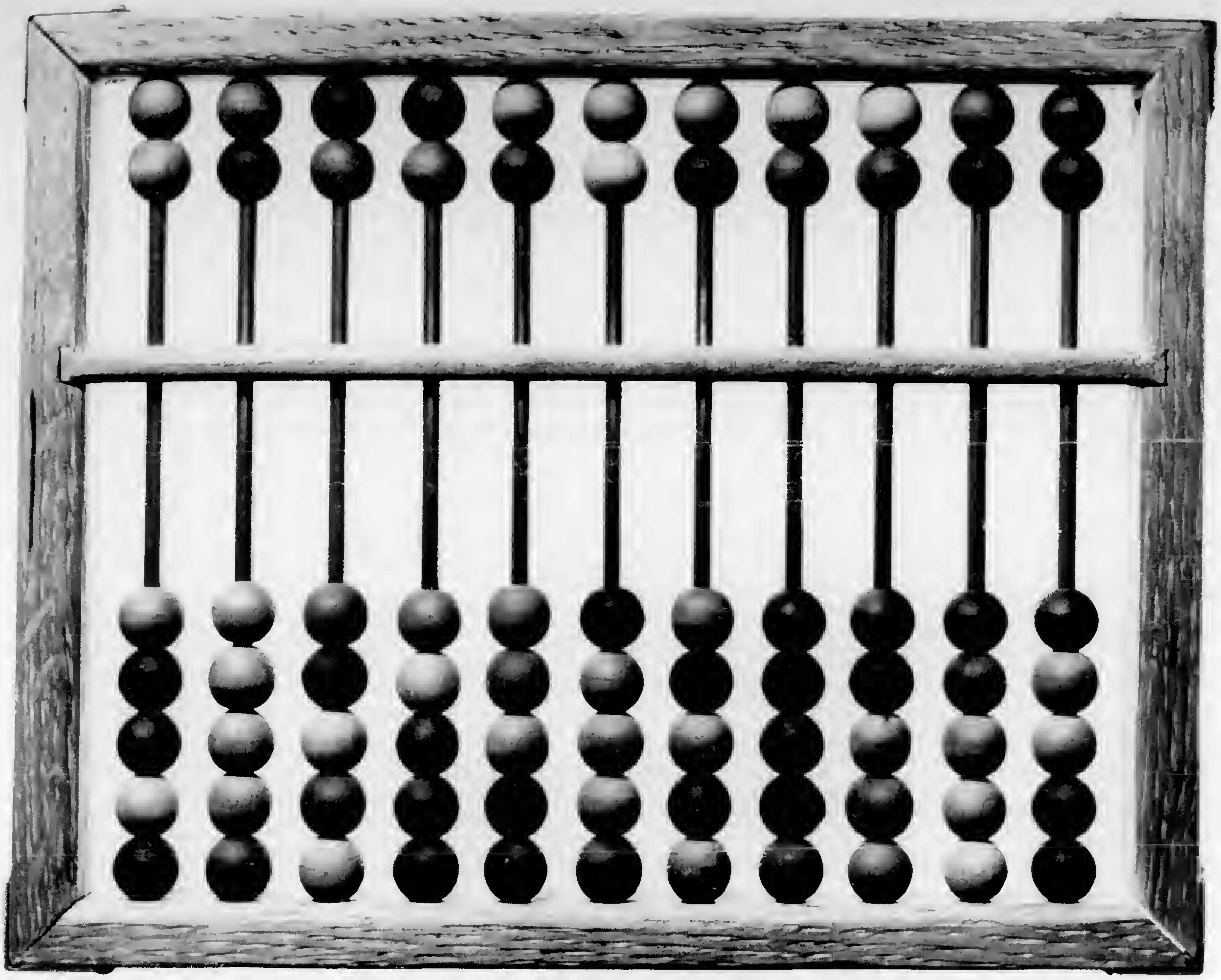
This problem which was faced by Oriental demonstrators closely resembles the one which now confronts the blind abacist. The Oriental solution is based upon the principle of increasing the size of the rods to create sufficient friction against the beads. An application of this principle has been used by the author in an attempt to adapt the simple bead frame for the use of the blind (Illus. G). The original wires were replaced by welding rod with a diameter which is slightly greater than that of the holes in the beads.



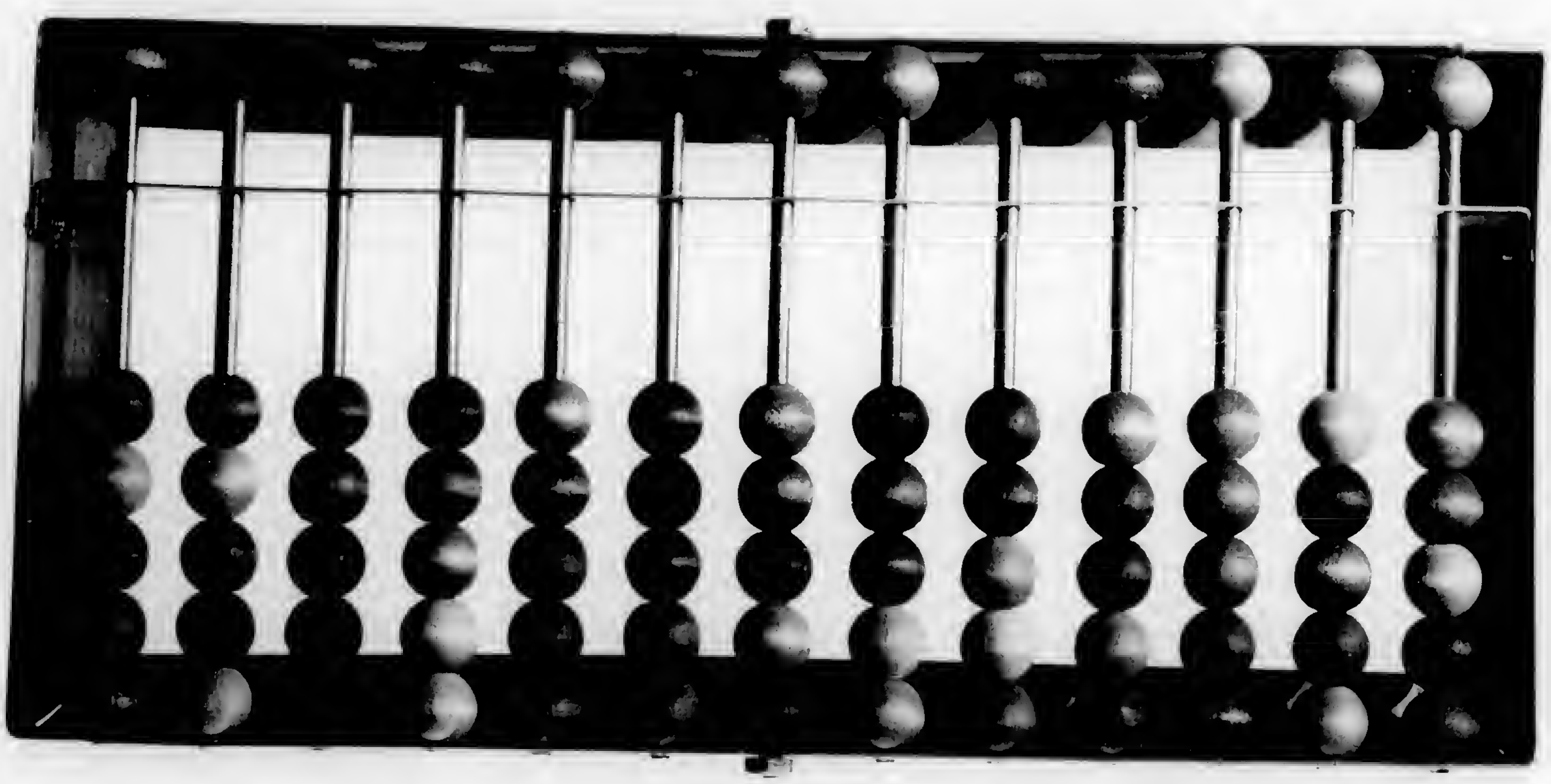
Illus. G - Adaptation of the Simple Bead Frame

The hole in each bead was then carefully enlarged until the desirable amount of friction existed between the rod and the bead. The fifth bead from each end was grooved to assist the blind operator in making rapid calculations. Welding rod was also used to construct models of the _swan-pan_ (Illus. H) and the _soroban_ (Illus. I). Wooden kindergarten beads were adjusted to the size of this welding rod. The frame of the _swan-pan_ was constructed of wood. Sheet brass was used instead of wood in constructing the frame of the _soroban_. Metal frames are stronger and less cumbersome than those which are made of wood. The three welding rod models of the abacus were tried out by students attending the California School for the Blind. Their criticisms were considered as basic suggestions to be followed in designing additional models. There was general agreement among the students that the counters should be stabilized, but they differed on the amount of friction that should be used. As the blind abacists became more skillful in the manipulation of the beads they usually found less friction to be desirable. Some believed that their tactual skill could be developed to the point where stabilization of the beads would cease to be necessary.

Another valuable suggestion which resulted from



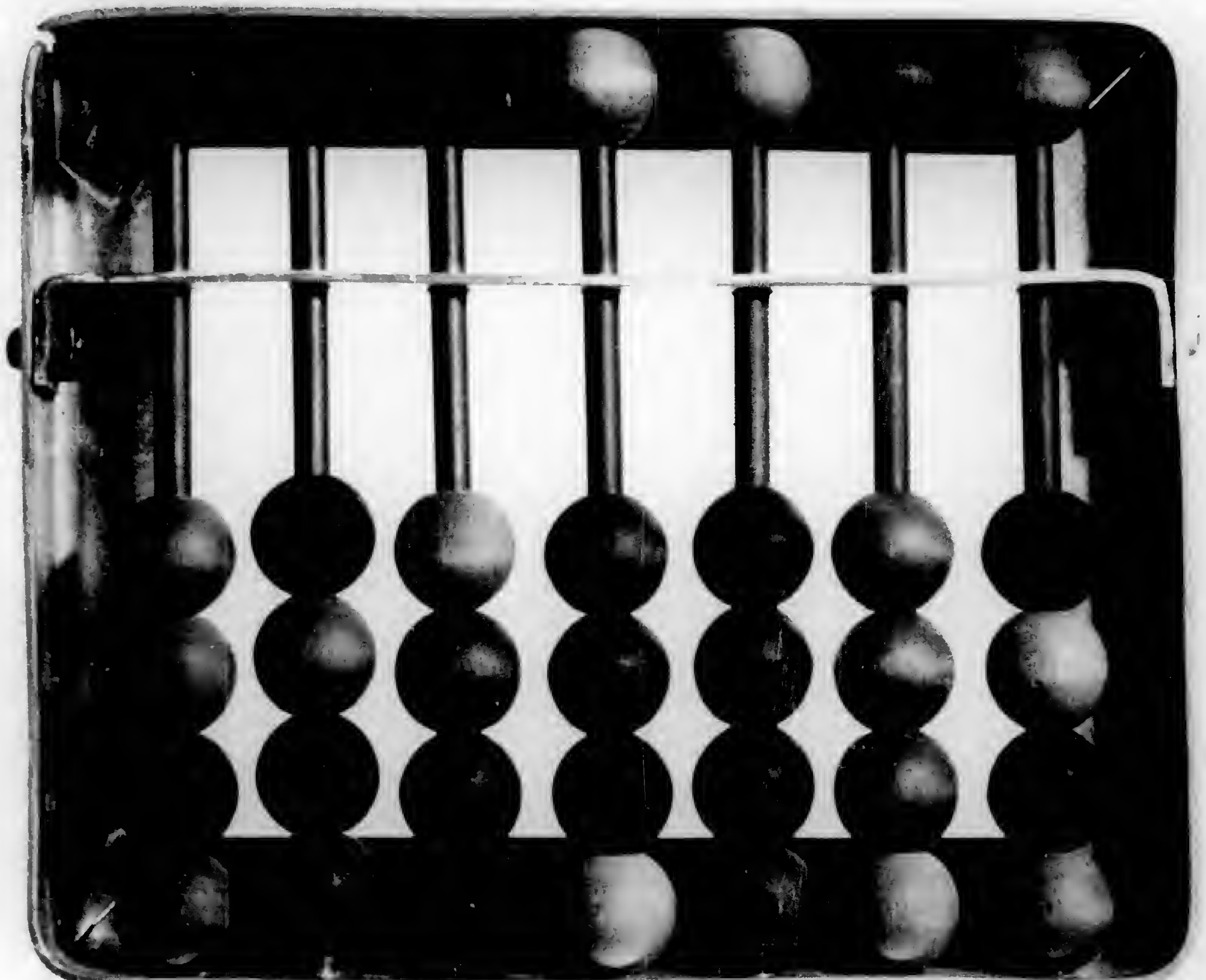
Illus. H - Adaptation of the swan-pan



Illus. I - Adaptation of the soroban

the preliminary tests of the three models was concerned with their sizes. These models were criticized most frequently on the grounds that they were too large to be carried about with convenience. A small portable model would serve the mathematical needs of the blind better than would the larger models. In order to reduce the size of the adapted abacus without impairing its utility it was necessary to modify the structural designs of both the beads and the frame. The changes which the Japanese had made in the structure of the Chinese swan-pan were advantageously used for this purpose. Thus, the number of beads required for each vertical rod was reduced from seven to five. The size of the frame was also significantly reduced. These are the suggested features which were applied in the construction of a pocket-sized model (Illus. J). Lightweight sheet aluminum was used in making the frame. In this frame were placed seven vertical rods each possessing five counters.

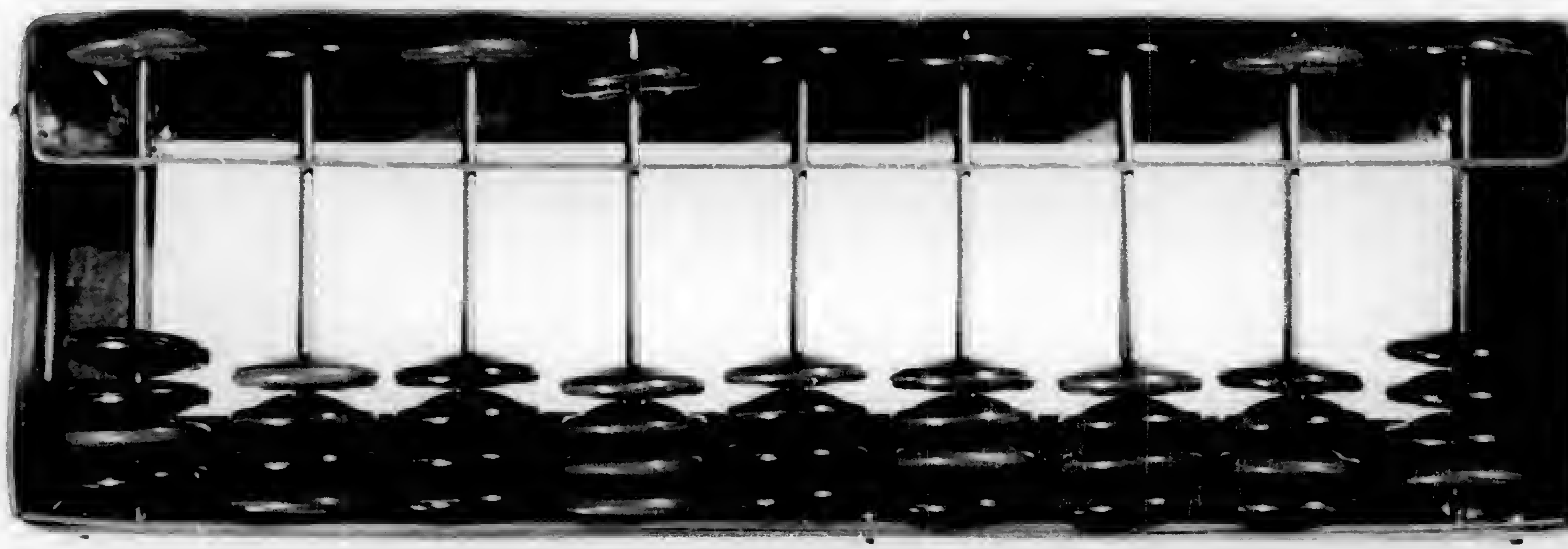
The blind students who tested the pocket-sized abacus found it to be more satisfactory for general use than the larger table models. It is small enough to be carried about in an inside coat pocket. There it will always be available to record numbers or to make computations. However, this pocket-sized model



Illus. J - Pocket-sized Adaptation of the <u>soroban</u>

was not accepted without criticism. The seven decimal places were found to be a limitation in certain mathematical processes. Therefore, it was recommended that nine places be provided instead of just seven. Also there continued to be disagreement concerning the amount of stabilization that should be applied to the beads. By this time it was obvious that the welding rod principle did not fully meet the needs resulting from individual differences in tactual dexterity.

In constructing the next model (Illus. K) an attempt was made to improve upon the welding rod principle which had been used for previous models. Instead of using a single rod for each decimal place the size of the rod was decreased sufficiently to permit the number to be doubled. The wooden kindergarten beads were replaced by shirt buttons. The two holes in each button provided the means by which they were mounted upon the double rods. A certain amount of tension could be applied to each button by spreading the rods farther apart than the distance between the centers of the two holes. This double rod principle required a new technique of counter manipulation. Each button had to be moved on a plain which was perpendicular to that of the rods. If the top of the button were pushed in either direction the angle of slide became so acute



Illus. K - Shirt Button Abacus (double rod principle)

that it prevented movement. This braking effect occurred in direct relationship to the tilting of the button against the double rods.

The shirt button abacus was not favorably received by the blind students who compared it with previous models. They appreciated the decreased width of the frame, but disapproved of the new fingering technique which was required. As a result of this unfavorable criticism further development of the model was discontinued. The idea of stabilization through structural modification of the parallel rods was also abandoned. Sliding kindergarten beads which had been carefully adjusted to welding rod seemed to be quite satisfactory for the use of beginners, but they lacked the degree of frictional adjustability which is required by individual differences among skilled operators. Therefore, the structural features of the sliding counters must be changed in a manner that will permit a greater degree of adjustability. The final area of research was, then, concerned primarily with the development of a new principle which would be an improvement upon that of the sliding counter.

The most promising substitute for the principle of the sliding counter which has been developed is one which possesses the characteristics of a lever of the

second class. The distance through which the effort arm of this lever passes can be made equivalent to that which is allowed for a sliding counter. The adoption of the lever principle eliminates not only the sliding beads, but also the parallel rods upon which they are mounted. Decimal places are represented by parallel grooves (Illus. L). The fulcrum of each lever is placed within a groove by attaching its lower end to a horizontal bar. This bar extends across the lower part of each groove in the frame. If the frame contains nine decimal places the horizontal bar will pass through the bases of nine levers. Four of these bars are placed below the horizontal crosspiece and one above it.

The operation of this lever abacus is a much simpler process than that required by previous models. The top of each lever is bevelled to facilitate rapid detection and manipulation. The levers are moved into position by pressing against their pointed ends with the fingers. Their position is stabilized by means of friction with the sides of the grooves in which they turn. Also there is some friction between the lever and the bar upon which it is mounted.

The model which has just been described does not possess adjustable levers. It was constructed for the purpose of determining whether or not the lever prin-

Illus. L - First Lever Abacus

ciple is a practical one. Students at the California School for the Blind prefer the lever abacus to any of the other models. Their support of this new principle has centered all experimentation upon the problem of the adjustable lever. The final model (Illus. M) illustrates the most satisfactory solution which has been developed for this problem. The frictional adjustment of the levers has been accomplished by splitting the base of each lever in such a way that a setscrew will tighten the split halves against their horizontal bar. These setscrews are adjusted by means of a screwdriver to suit the tactual needs of any blind operator. This type of lever eliminates the necessity for constructing a frame with vertical grooves. The decimal places are maintained along the horizontal bars by placing small spacers between each vertical row of five levers.

Dr. R. S. French, superintendent of the California School for the Blind, has approved a plan for the construction of the lever type abacus in sufficient quantity to permit experimental use in various classroom situations. A bar aluminum alloy will be used instead of wood for both levers and frame. Each abacus will possess nine decimal places with a sliding decimal point attached to the frame. Also it has been recommended that a means should be provided by which a section of nine decimal

Illus. M - The Adjustable Lever Model of the Abacus

places may be supplemented by the addition of other sections. This provision will be of great assistance in complex computation processes. It is hoped that the use of these experimental models will stimulate suggestion and criticism which will be of value for improving their mathematical usefulness to the blind.

CHAPTER IV

HOW TO OPERATE THE ABACUS

The purpose of this chapter is to illustrate how the abacus may be adapted to the various methods of computation which are now being taught to the blind. This instrument is capable of serving the mathematical needs of the blind as efficiently as pencil and paper serves those of the sighted. It is proposed as a mechanical aid to promote the attainment of more rapid and accurate numerical calculation. As a mechanical aid the abacus relieves the mind of the memory work which tends to distract it from the computational process itself. However, its operation is not automatic as are the modern adding and calculating machines. Therefore, its instructional values and mathematical potentialities are not limited by existing differences in teaching methods.

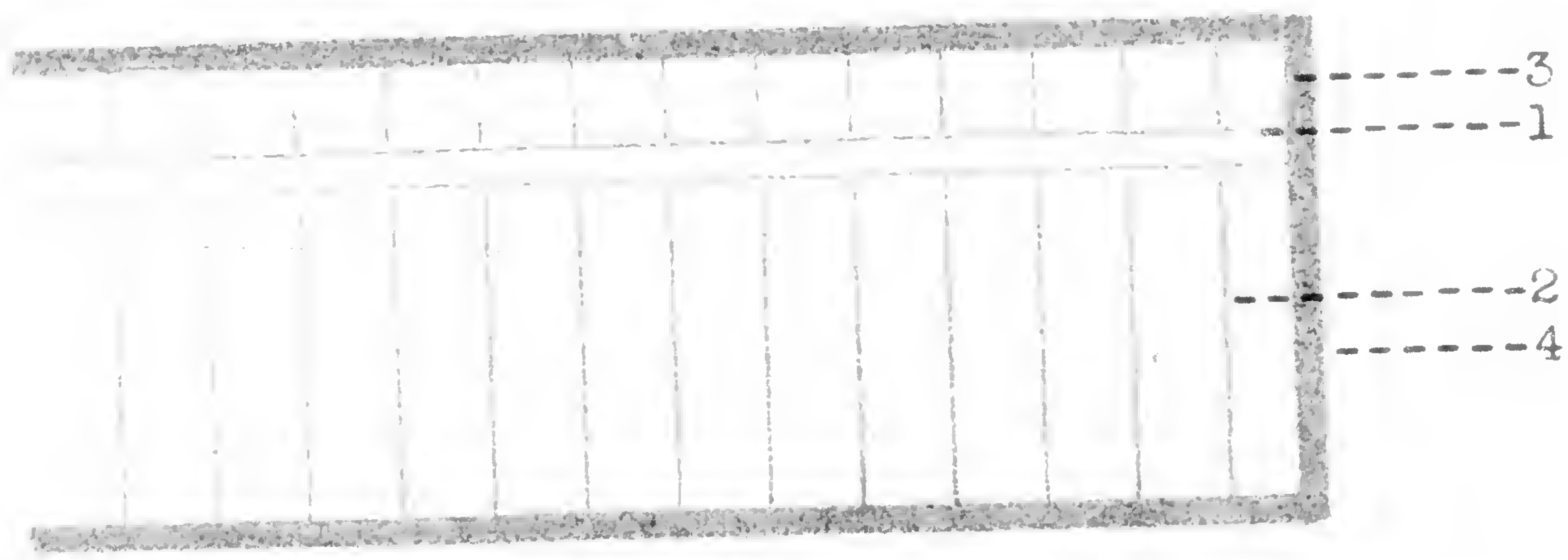
Each decimal place of the abacus is represented by a group of sliding counters. Numbers which have been placed upon the abacus can be changed or removed merely by adjusting the position of the counters. If these counters are to be used efficiently the operator must possess a basic knowledge of numerical relationships. This basic knowledge results from concrete

demonstrations and experiences rather than from a program of endless repetition. Hayes (9:101) emphasized this point when he stated:

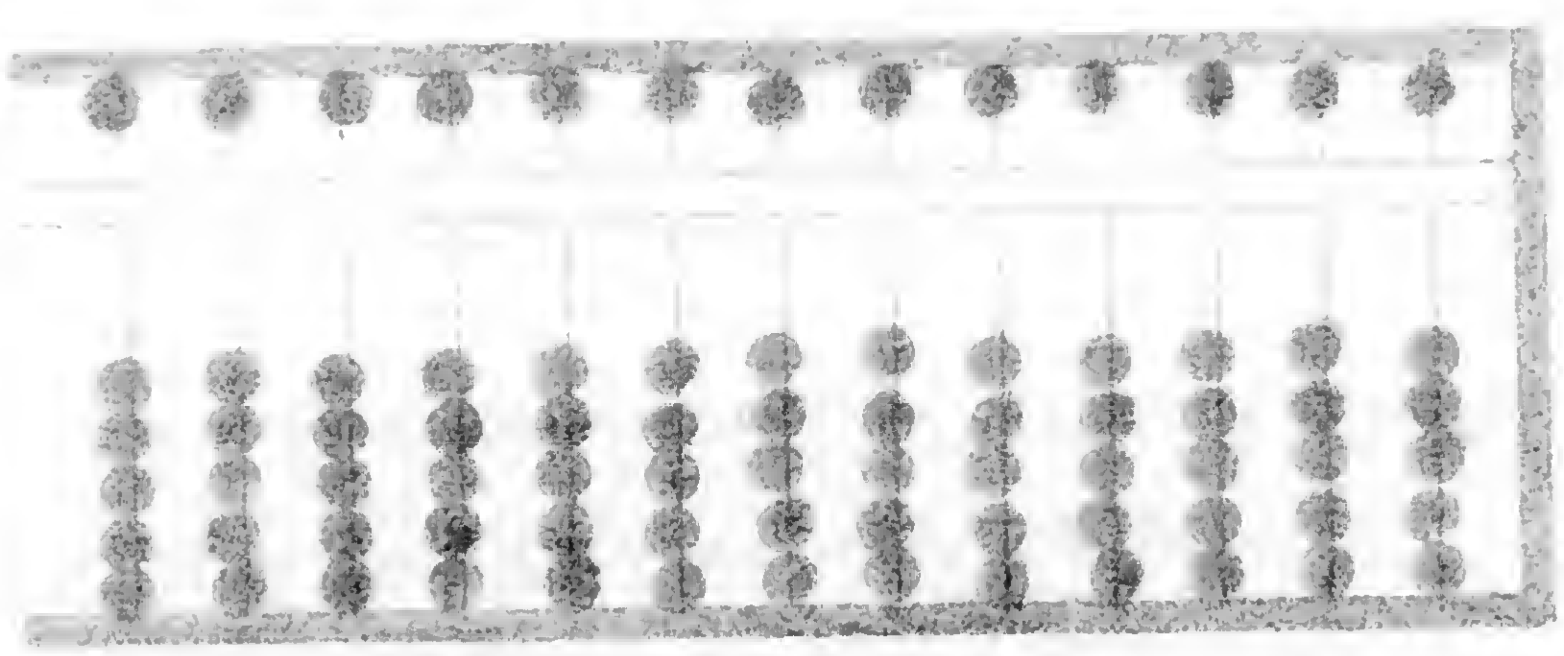
> "understanding rather than drill should be the central purpose."

The Frame and Its Counters

For illustrative purposes the Japanese soroban has been selected as the most representative form of the modern abacus. Yoshino (30:1) names the parts of this instrument as follows:



1 - the crosspiece
2 - an upright
3 - the upper part, or Heaven
4 - the lower part, or Earth



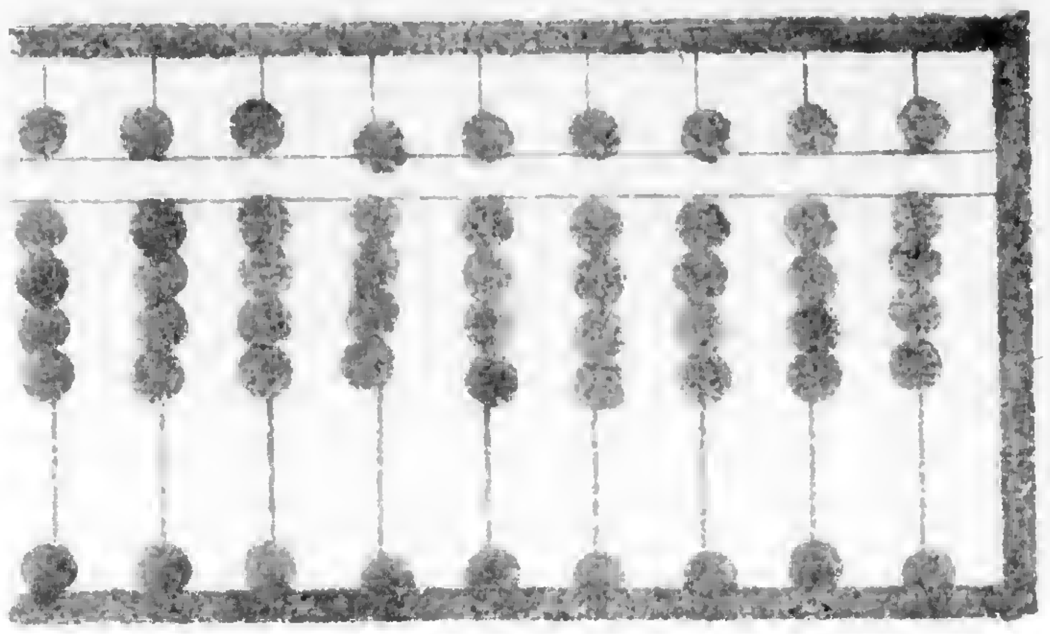
The Starting Position for the Counters

The counters are mounted upon the uprights and constitute the moving parts of the abacus. They are located on both sides of the horizontal crosspiece. Those in the upper part are known as heaven counters and those in the lower part as earth counters. A counter in heaven has the value of five. A counter in earth has the value of one. Therefore, the numerical value of each heaven counter is equal to that of five earth counters.

In describing the various positions and movements of the counters it is advisable to define for the reader any special terminology which must be used. Therefore, the following definitions are presented:

1. To raise means to move up earth counters against the crosspiece.
2. To lower five means to move down a heaven counter against the crosspiece.
3. To cancel means to take counters away from the middle crosspiece.
4. To forward ten means to move up one earth counter on the next upright to the left.

The numerical capacity of an abacus which possesses nine uprights is 999,999,999.



The counters are considered as numbers only when they have been moved against the horizontal crosspiece. When the counters on an upright are canceled the number in that decimal place becomes a zero.

321

300

The position of the decimal point is usually determined arbitrarily by the person who is using the abacus. If a decimal point is not desired, the ones' or units' place lies at the right-hand end. The numbers from one to twenty are shown on the abacus as follows:

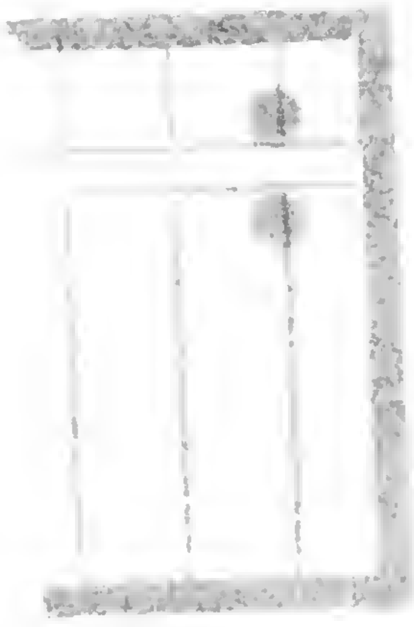
1

2

3

4

5



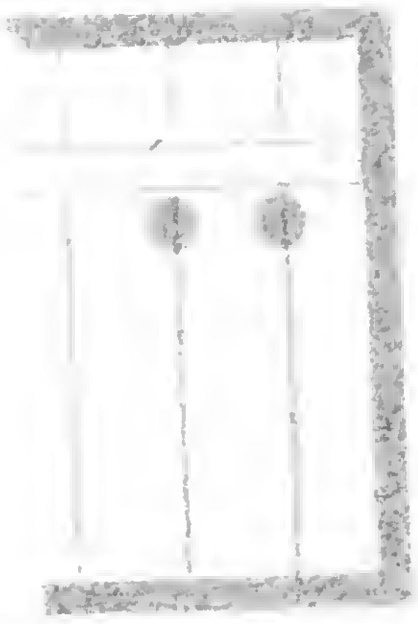
6

7

8

9

10



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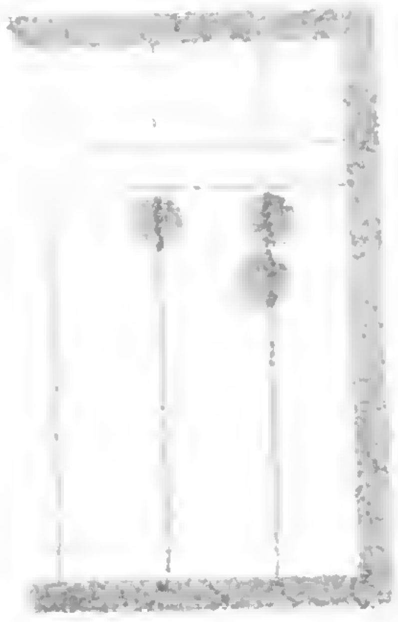

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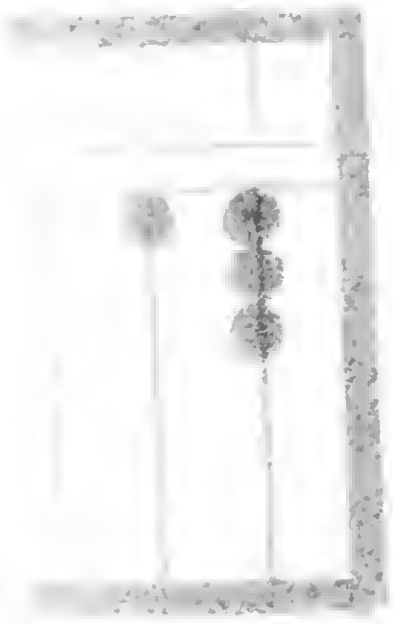

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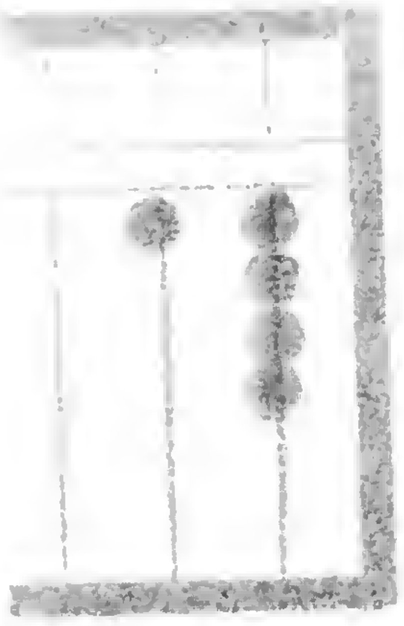

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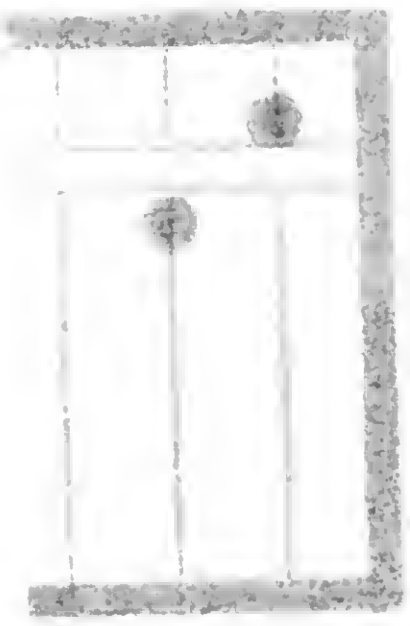

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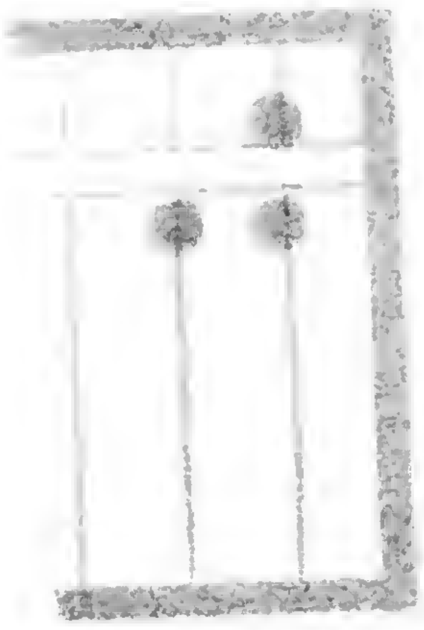

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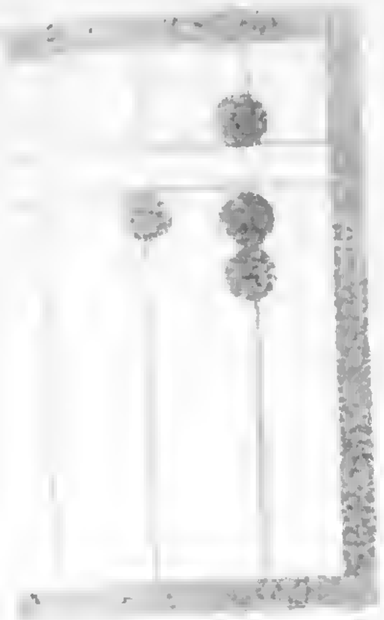
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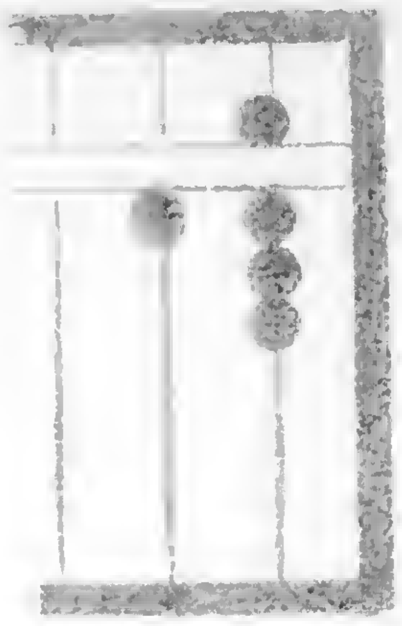
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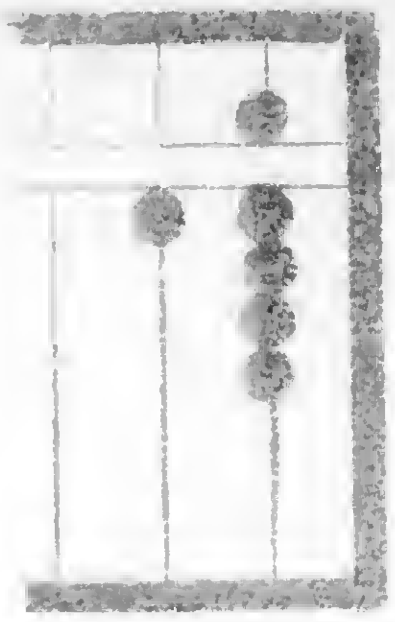
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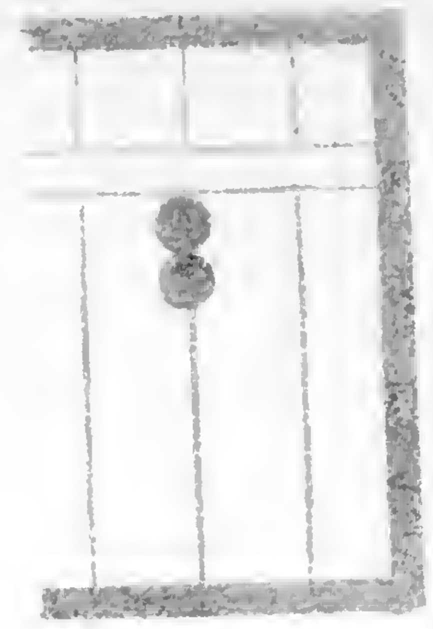

20

123,456,789 contains all of the basic combinations which are used for placing larger numbers on the abacus.

123,456,789

Addition

The process of addition regardless of the method employed requires a knowledge of certain basic number combinations which are frequently referred to as "number facts". The following table is memorized by most beginners:

1 plus 9 = 10

2 plus 8 = 10

3 plus 7 = 10

4 plus 6 = 10

5 plus 5 = 10

6 plus 4 = 10

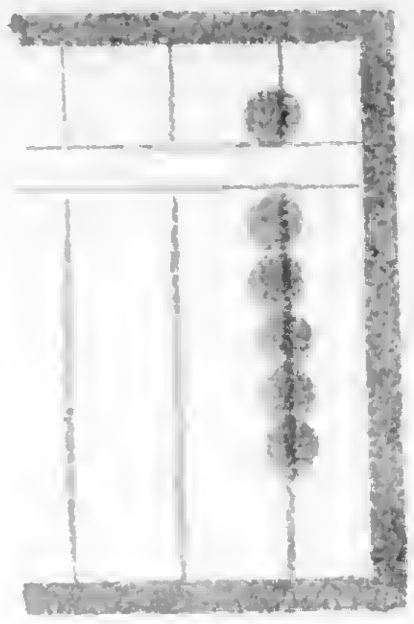
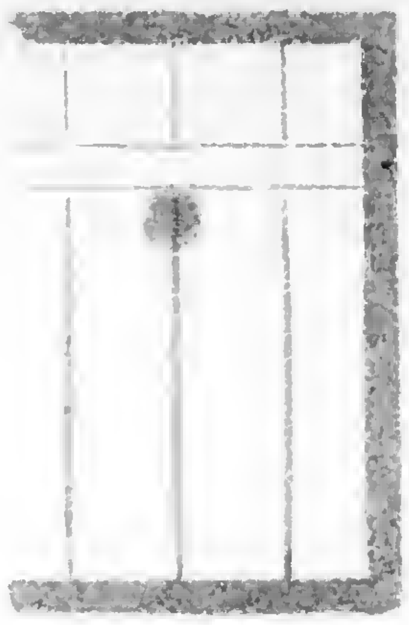
7 plus 3 = 10

8 plus 2 = 10

9 plus 1 = 10

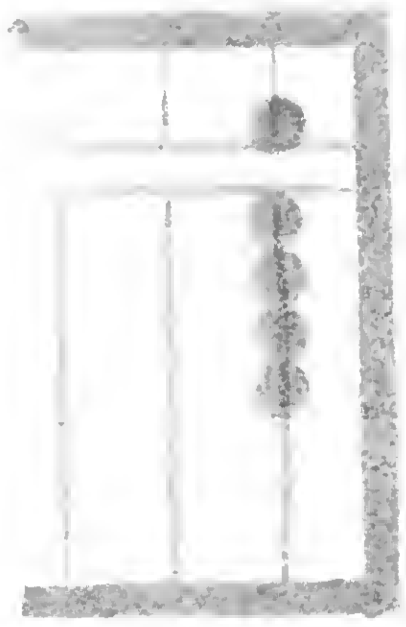
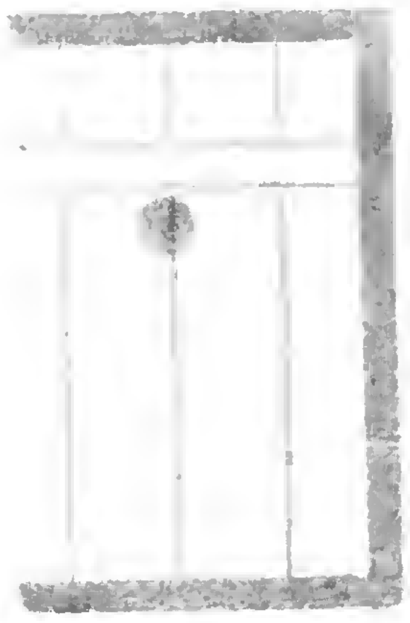
The simple bead frame (Illus. G) is the most numerically concrete means by which the basic combinations presented in the previous table may be demonstrated. The Japanese soroban introduces an abstraction in the form of a heaven counter which has the value of five earth counters. The heaven counter is one of the various features of the modern Oriental abacus which has made it such an efficient computing device.

The heaven counter and five earth counters possess the total value of ten in each decimal place. (Example A) However, ten is a two-place figure and is represented by raising an earth counter in tens' place. (Example B)

Example A   Example B

The basic combinations of ten are placed on the soroban as illustrated:

1 plus 9 = 10

2 plus 8 = 10

3 plus 7 = 10

4 plus 6 = 10

5 plus 5 = 10

6 plus 4 = 10

7 plus 3 = 10

8 plus 2 = 10

9 plus 1 = 10

Kwa Tak Ming (21:11) presents seventeen guides or hints which show how to add numbers on the abacus. The first part of each guide contains the number to be added and the last part tells how it should be done.

1. One; lower five, cancel four

4 plus 1 = 5

2. Two; lower five, cancel three

4 plus 2 = 6

3. Three; lower five, cancel two

4 plus 3 = 7

4. Four; lower five, cancel one

4 plus 4 = 8

5. One; cancel nine, forward ten

9 plus 1 = 10

6. Two; cancel eight, forward ten

9 plus 2 = 11

7. Three; cancel seven, forward ten

9 plus 3 = 12

8. Four; cancel six, forward ten

9 plus 4 = 13

9. Five; cancel five, forward ten

9 plus 5 = 14

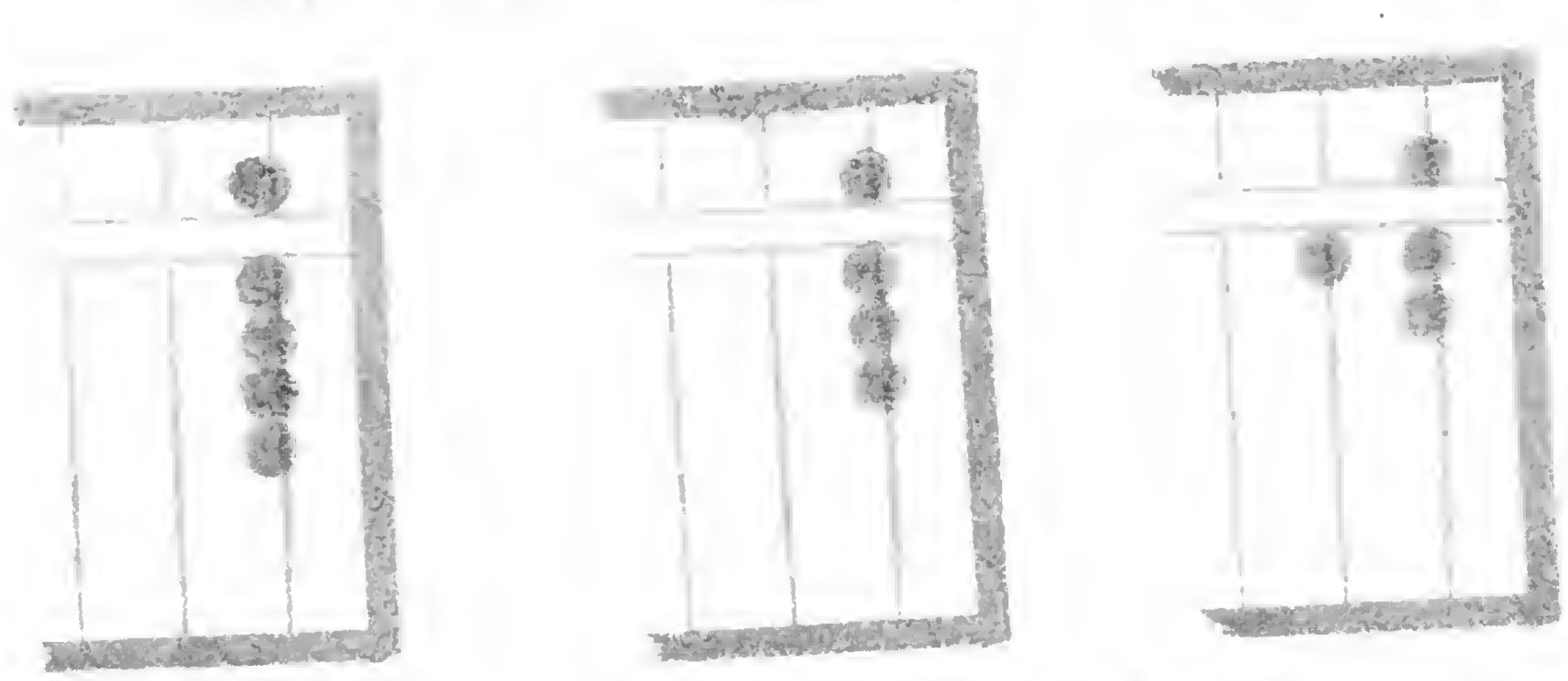
10. Six; cancel four, forward ten

9 plus 6 = 15

11. Seven; cancel three, forward ten

9 plus 7 = 16

12. Eight; cancel two, forward ten



9 plus 8 = 17

13. Nine; cancel one, forward ten

9 plus 9 = 18

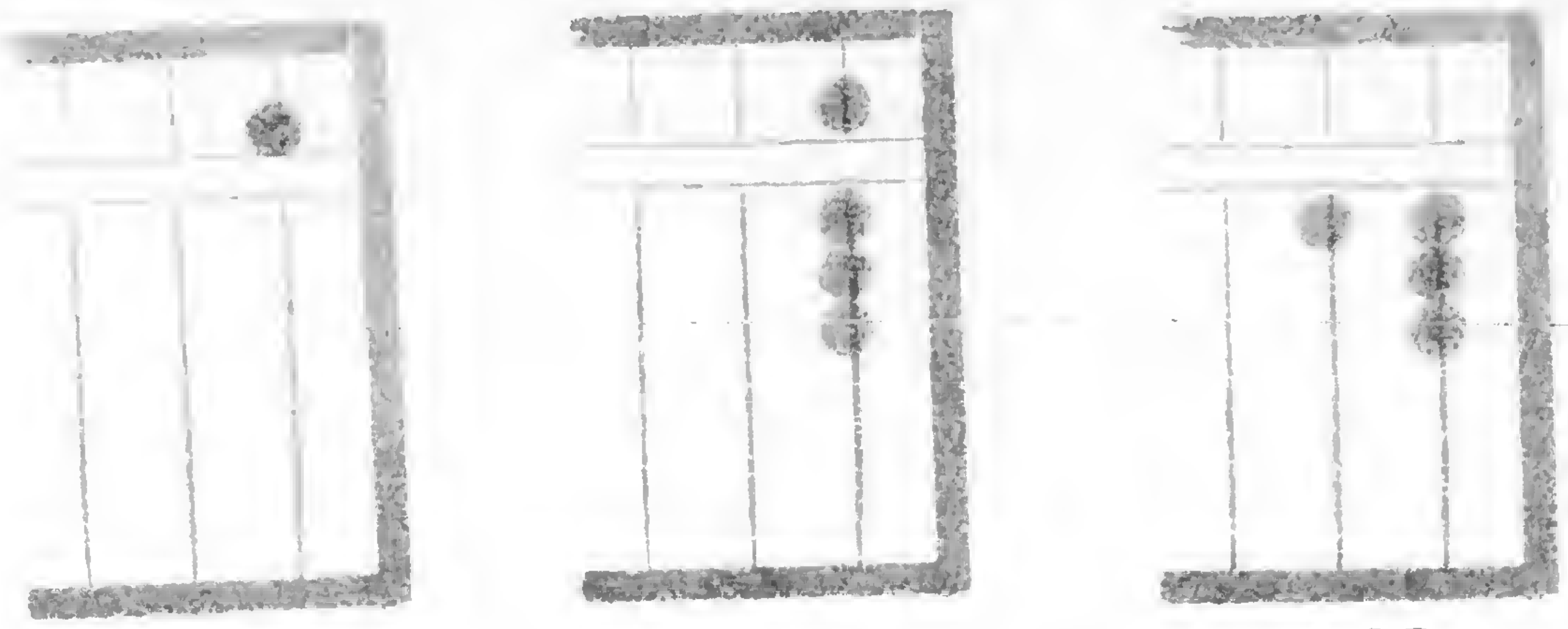
14. Six; raise one, cancel five, forward ten

5 plus 6 = 11

15. Seven; raise two, cancel five, forward ten

5 plus 7 = 12

16. Eight; raise three, cancel five, forward ten



5 plus 8 = 13

17. Nine; raise four, cancel five, forward ten

5 plus 9 = 14

The two most commonly used methods for addition are:

I. Adding from left to right

II. Adding from right to left

The first method begins with the highest or left-hand place.

Example A

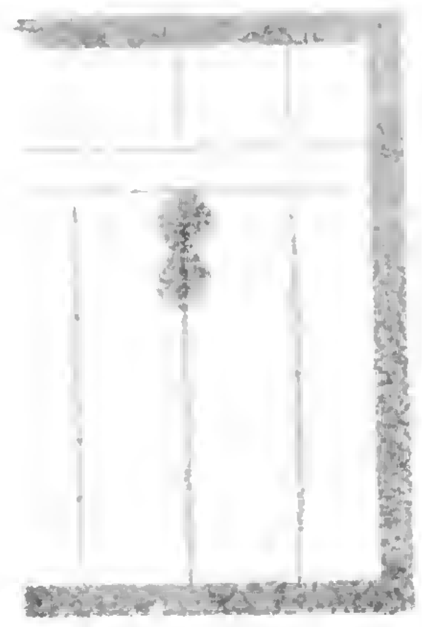
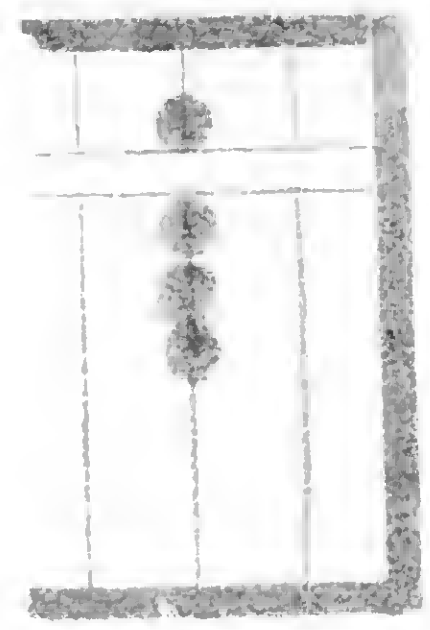
$$\begin{array}{r} 67 \\ +\,22 \\ \hline 89 \end{array}$$

Variation 1:

a. 60 + 20 = 80
b. 80 + 7 = 87
c. 87 + 2 = 89 (sum)

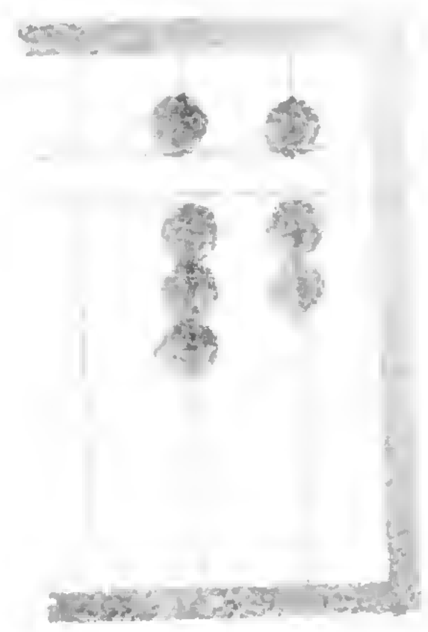
<u>On the abacus</u>:

a. 60 (lower five, and raise one in tens' place) + 20 (raise two) = 80

60 plus 20 = 80

b. 80 + 7 (lower five and raise two in units' place) = 87



87

c. 87 + 2 (raise two) = 89 (sum)

89

Variation 2:

a. 67 + 20 = 87
b. 87 + 2 = 89 (sum)

On the abacus:

a. 67 + 20 (raise two earth counters in tens' place) = 87

67 plus 20 = 87

b. 87 + 2 (raise two) = 89 (sum)

89

Example B

$$\begin{array}{r} 52 \\ +\,39 \\ \hline 91 \end{array}$$

Adding from left to right:

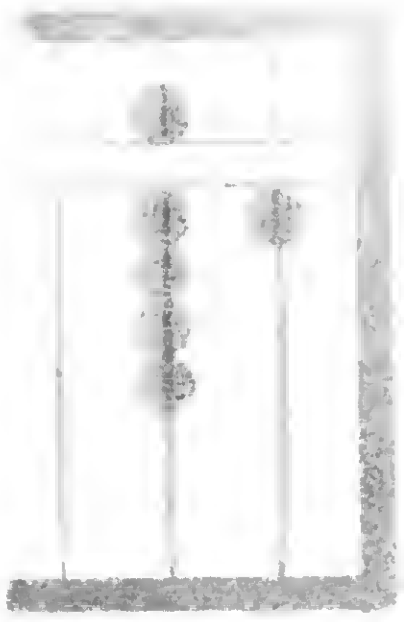
a. 52 + 30 = 82
b. 82 + 9 (one and ten to carry) = 91

<u>On the abacus:</u>

a. 52 + 30 (raise three) = 82

52 plus 30 = 82

b. 82 + 9 (cancel one; forward ten) = 91 (sum)



91

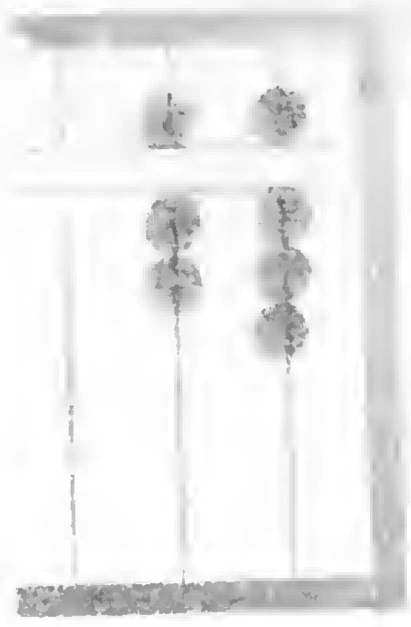
<u>Example C</u>

$$\begin{array}{r} 78 \\ +\,63 \\ \hline 141 \end{array}$$

a. 78 + 60 (three and ten to carry) = 138
b. 138 + 30 (one and ten to carry) = 141 (sum)

<u>On the abacus:</u>

a. 78 + 60 (raise one; cancel five; forward ten) = 138

78 plus 60 = 138

b. 138 + 3 (cancel seven; forward ten) = 141 (sum)

141

The second method begins with the units or righthand place.

Example A

```
    67
  + 22
    89 (sum)
a.  7 + 2 = 9
b.  60 + 20 = 80
      Sum = 89
```

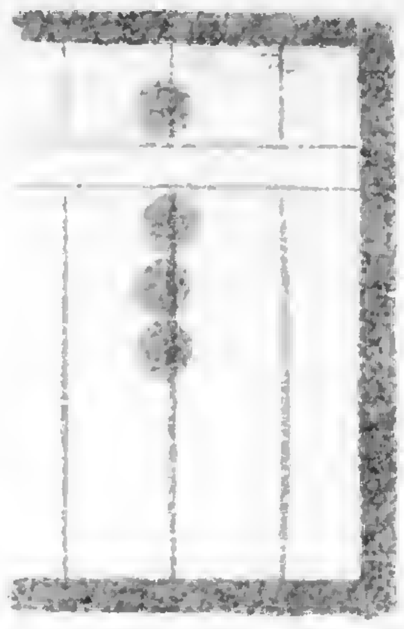
On the abacus:

a. 7 + 2 (raise two) = 9

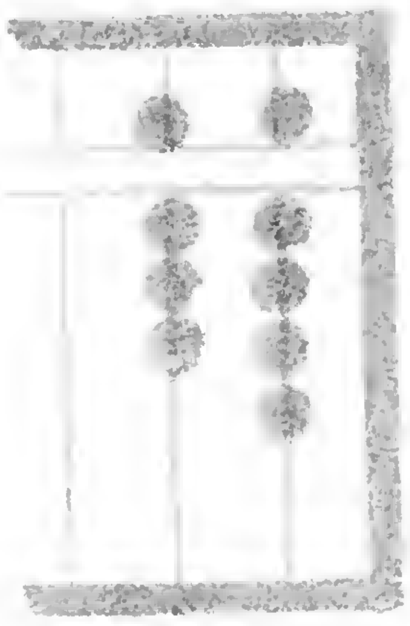
7 plus 2 = 9

b. 60 + 20 (raise two) = 80

60 plus 20 = 80

SUM = 89



89

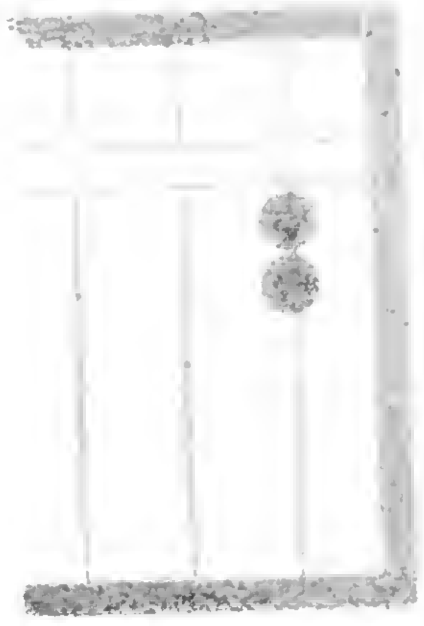
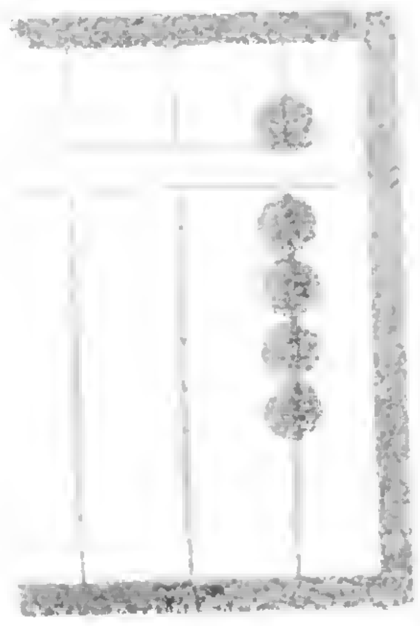
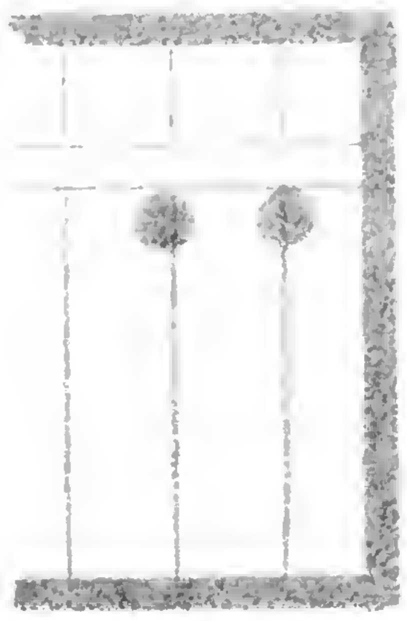
Example B

$$\begin{array}{r} 52 \\ +\,39 \\ \hline 91 \end{array}$$

a. 2 + 9 = 11 (one and ten to carry)
b. 50 + 30 + 10 (carried over) = 90
Sum = 91

On the abacus:

a. 2 + 9 (cancel one; forward ten) = 11

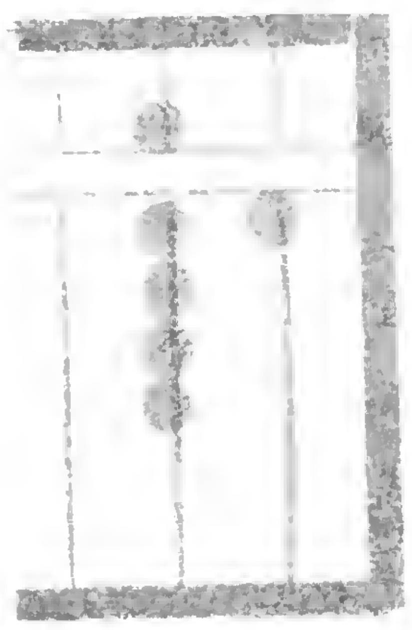
  

2 plus 9 = 11

b. 10 (carried) and 50 (lower five) + 30 (raise three) = 90

10 (carried) and 50 + 30 = 90

SUM = 91



91

Example C

$$\begin{array}{r} 78 \\ +63 \\ \hline 141 \end{array}$$

a. 8 + 3 = 11 (one and ten to carry)
b. 70 + 60 + 10 (carried over) = 140
Sum = 141

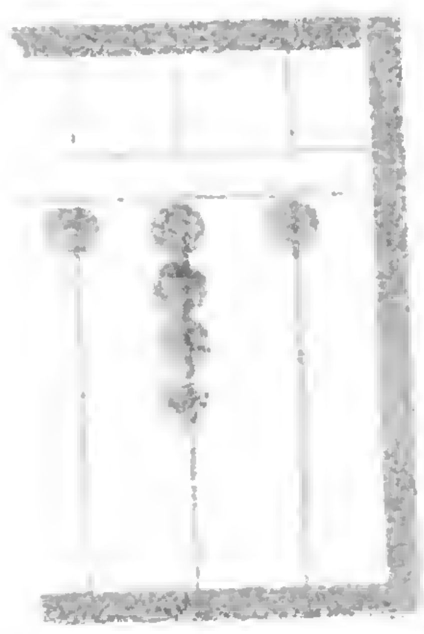
On the abacus:

a. 8 + 3 (cancel seven; forward ten) = 11

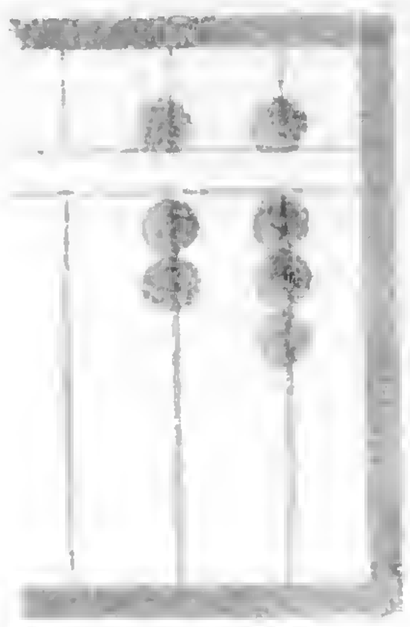
8 plus 3 = 11

b. 10 (carried) and 70 (lower five; raise two)
+ 60 (raise one; cancel five; forward ten)
140

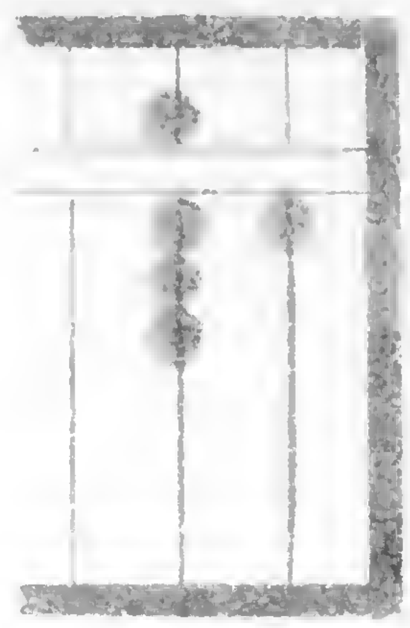
SUM=141



The following example illustrates a more popular variation of this method as it is performed on the abacus:



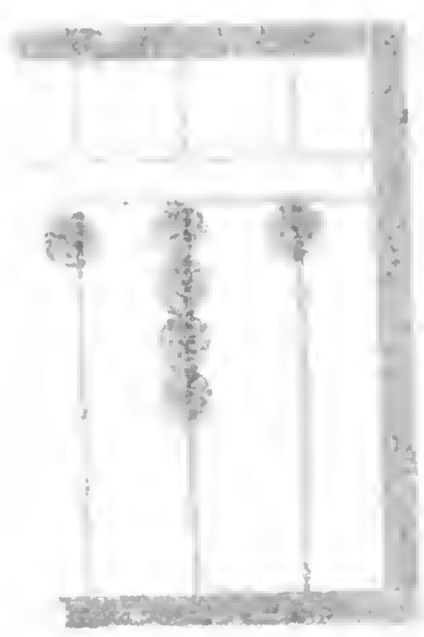
78



81

78 + 3 (cancel seven; forward ten) = 81
Add 60 (raise one; cancel five; forward ten)

SUM = 141

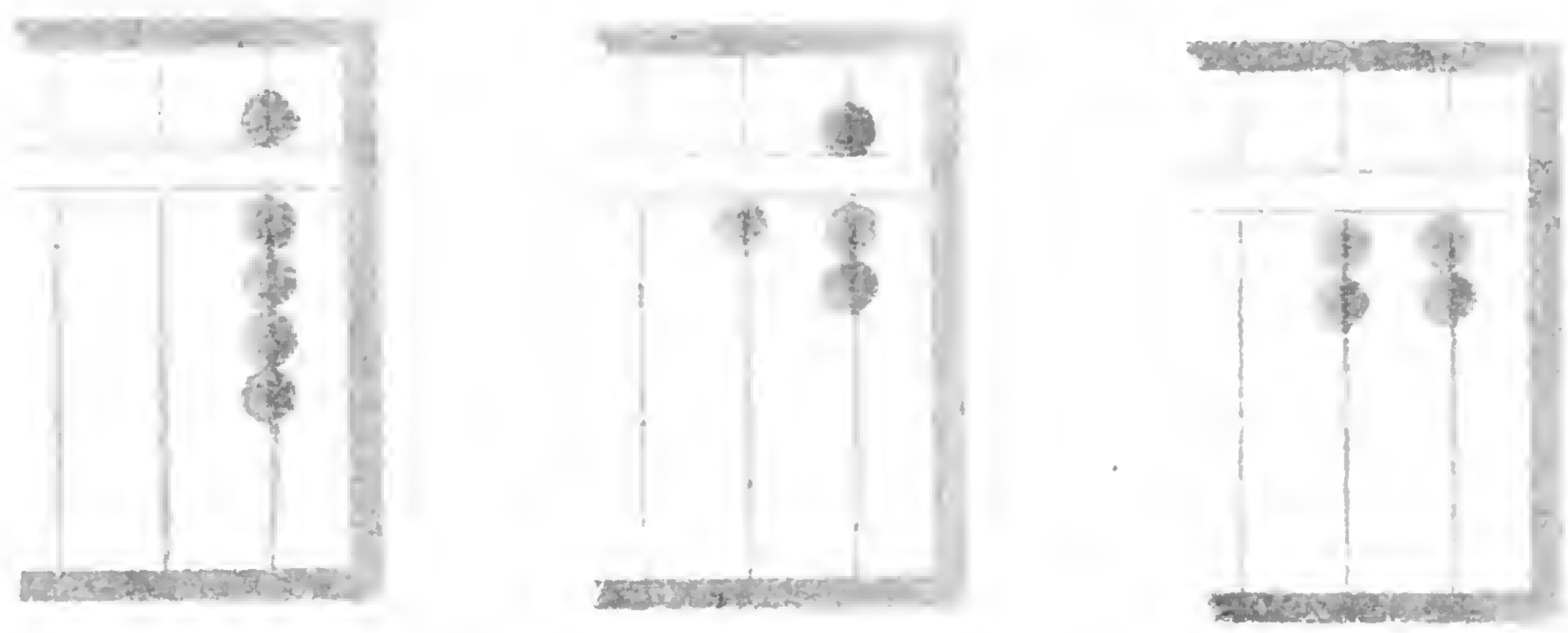


141

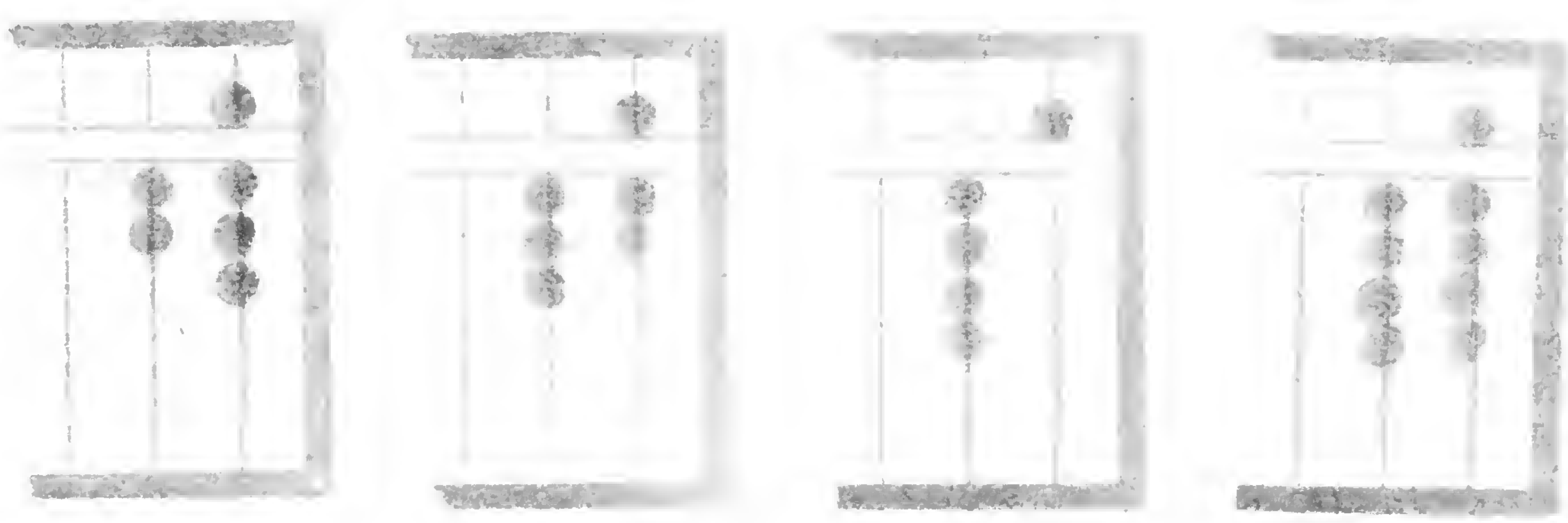
Complex problems in addition are solved on the abacus in the same manner as are the relatively simple ones which have been demonstrated in this section. It should be pointed out that the abacus possesses a unique advantage in adding long columns of numbers. As each number is recorded it is thereby added to the preceding sum.

Example:

$$\begin{array}{r} 9 \\ 8 \\ 5 \\ 6 \\ 9 \\ 8 \\ \underline{4} \\ 49 \end{array}$$



9 + 8 = 17 + 5 = 22 + 6 =



28 + 9 = 37 + 8 = 45 + 4 = 49

Subtraction

The process of subtraction on the abacus is the reverse ofthat used for addition. In order to solve a problem of this type the beginner must memorize the following table: (30:54)

10 - 1	9
10 - 2	8
10 - 3	7
10 - 4	6
10 - 5	5
10 - 6	4
10 - 7	3
10 - 8	2
10 - 9	1

As in addition certain guides have been formulated which may be useful in subtracting numbers on the abacus. (21:15) The first part of each guide contains the number to be subtracted and the last part tells how it should be done.

1. One; cancel five, return four

5 - 1 = 4

2. Two; cancel five, return three

5 - 2 = 3

3. Three; cancel five, return two

5 - 3 = 2

4. Four; cancel five, return one

5 - 4 = 1

5. One; cancel ten, return nine

10 - 1 = 9

6. Two; cancel ten, return eight

10 - 2 = 8

7. Three; cancel ten, return seven

10 - 3 = 7

8. Four; cancel ten, return six

10 - 4 = 6

9. Five; cancel ten, return five

10 - 5 = 5

10. Six; cancel ten, return four

10 - 6 = 4

11. Seven; cancel ten, return three

10 - 7 = 3

12. Eight; cancel ten, return two

10 - 8 = 2

13. Nine; cancel ten, return one

10 - 9 = 1

14. Six; cancel ten, return five, cancel one

14 - 6 = 8

15. Seven; cancel ten, return five, cancel two

14 - 7 = 7

16. Eight; cancel ten, return five, cancel three

14 - 8 = 6

17. Nine; cancel ten, return five, cancel four

14 - 9 = 5

The three most commonly used methods of subtraction are:

I. Subtracting from left to right

II. Subtracting from right to left

III. Subtracting by adding

The first method begins with the highest or left-hand place.

Example A

$$\begin{array}{r} 89 \\ \underline{-34} \\ 55 \end{array}$$

1. Subtracting from left to right

a. 80 - 30 = 50
b. 9 - 4 = 5
Remainder = 55

<u>On the abacus</u>:

a. 80 - 30 (cancel three tens) = 50

80 - 30 = 50

b. 9 - 4 (cancel four units) = 50

9 - 4 = 5

Remainder = 55

Example B

$$\begin{array}{r} 72 \\ -58 \\ \hline 14 \end{array}$$

a. 70 - 50 = 20
b. 8 cannot be taken away from 2; borrow 10
c. 20 - 10 = 10
d. 10 + 2 = 12
e. 12 - 8 = 4
Remainder = 14

On the abacus:

a. 72 - 50 (cancel heaven counter in tens' place) = 22

b. 22 - 8 (cancel ten, raise two) = 14
Remainder = 14

Example C

$$\begin{array}{r} 123 \\ -\ 59 \\ \hline 64 \end{array}$$

a. 120 - 50 = 70
b. 9 cannot be taken from 3; borrow 10
c. 70 - 10 = 60
d. 10 + 3 = 13
e. 13 - 9 = 4
Remainder = 64

On the abacus:

a. 123 - 50 (cancel ten in the hundreds' place; lower five in tens' place) = 73
b. 73 - 9 (cancel ten; raise one) = 64
Remainder = 64

The second method begins with the units or right-hand place.

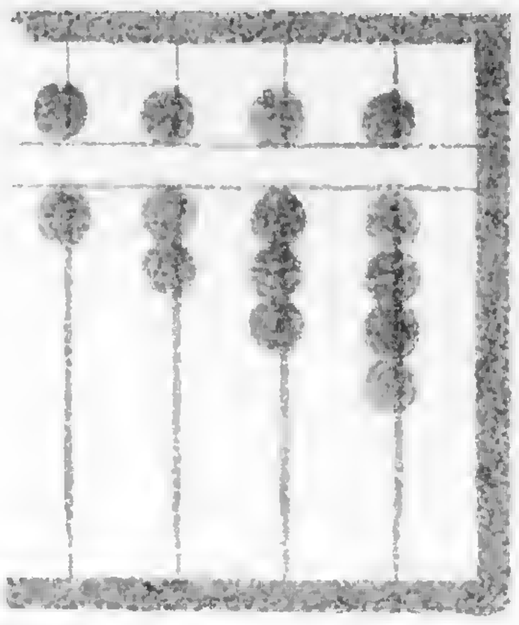
Example A

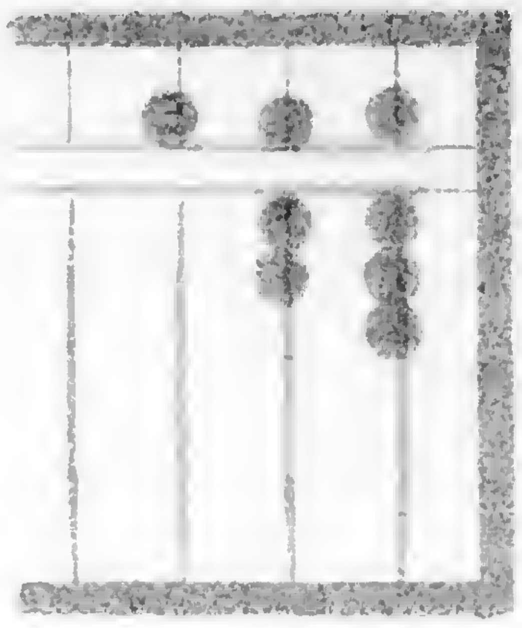
$$\begin{array}{r} 6{,}789 \\ -\ \ 578 \\ \hline 6{,}211 \end{array}$$

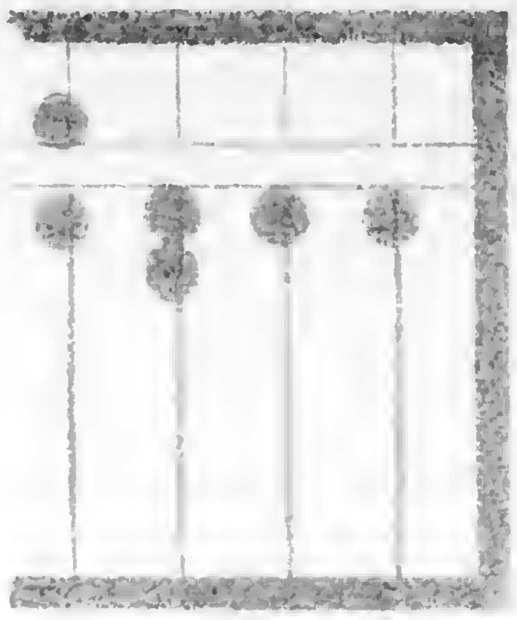
a. 9 - 8 = 1
b. 80 - 70 = 10
c. 700 - 500 = 200
d. 6,000 - 0 = 6,000
Remainder = 6,211

On the abacus:

a. 9 - 8 (cancel three earth counters and the heaven counter in units' place) = 1
b. 80 - 70 (cancel two earth counters and the heaven counter in tens' place) = 10
c. 700 - 500 (cancel the heaven counter in hundreds' place) = 200
Remainder = 6,211







6,789 - 578 = 6,211

Example B

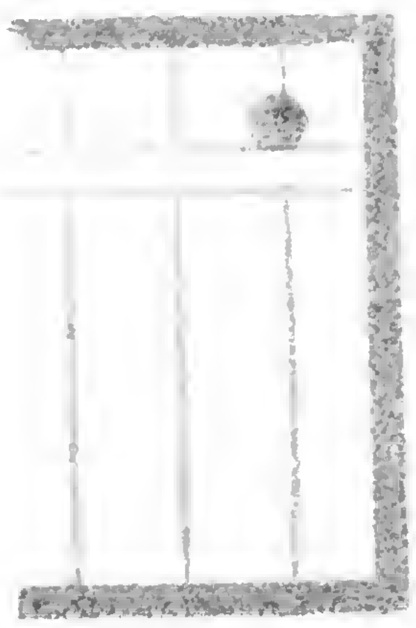
```
 164
- 85
  79
```

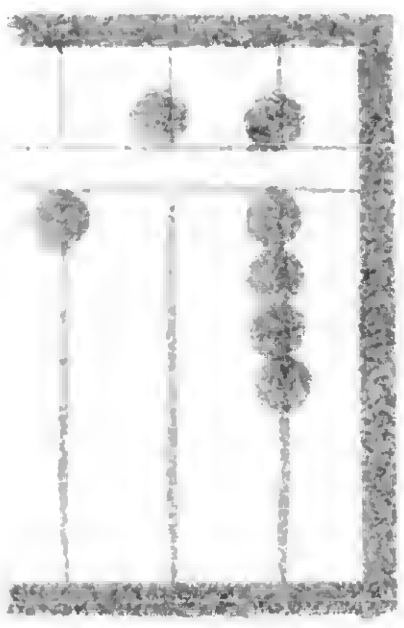
a. 5 cannot be taken from 4; borrow 10
60 - 10 = 50
10 + 4 = 14
14 - 5 = 9

b. 150 - 80 = 70
Remainder = 79

On the abacus:

a. 164 - 5 (cancel ten; lower five) = 159





164 - 5 = 159

b. 159 - 80 (cancel 100; raise twenty) = 79

159 - 80 = 79

The third method obtains the remainder by determining how much must be added to the subtrahend to equal the amount of the minuend. The number of steps to be used in this process is a matter of individual preference.

Example A

$$\begin{array}{r} 247 \\ -\ \ 79 \\ \hline 168 \end{array}$$

a. 79 + 1 = 80
b. 80 + 20 = 100
c. 100 + 100 = 200
d. 200 + 40 = 240
e. 240 + 7 = 247

Therefore: 1 + 20 + 100 + 40 + 7 = 168 (remainder)

On the abacus:

The function of the abacus with this method of subtraction depends upon the mathematical ability of the operator. In most cases it would be used to add up the steps which constitute the final remainder. Thus, the remainder for Example A is obtained by adding the following steps:

1 + 20 = 21 + 100 = 121

121 + 40 = 161 + 7 = 168

Example B

$$\begin{array}{r} 12{,}345 \\ -\ \underline{6{,}789} \\ 5{,}556 \end{array}$$

a. 6,789 + 11 = 6,800
b. 6,800 + 5,000 = 11,800
c. 11,800 + 500 = 12,300
d. 12,300 + 45 = 12,345
Therefore: 11 + 5,000 + 500 + 45 = 5,556 (remainder)

On the abacus:

Follow the same procedure as in Example A.

In view of the fact that there are several methods for addition and subtraction a certain degree of flexi-

bility is necessary for the recommendations which concern the use of the abacus. Teachers and students should be encouraged to use this instrument in the manner which most efficiently satisfies their individual needs. For instance, if the subtrahend of a subtraction problem is larger than the minuend it may be sufficient to place the subtrahend on the abacus and to take away the minuend. It should then be assumed that the remainder is preceded by a minus sign.

Multiplication

The process of multiplication is often considered as repeated addition. It is this concept upon which the following multiplication table is based:

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	47	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

There are several possible methods which may be used in multiplication, but only two of these are in general use with the blind. These two methods are:

I. Multiplying from right to left (multiplicand as well as multiplier)

II. Multiplying from left to right (multiplicand as well as multiplier)

The first method is the one which is usually employed with pencil and paper.

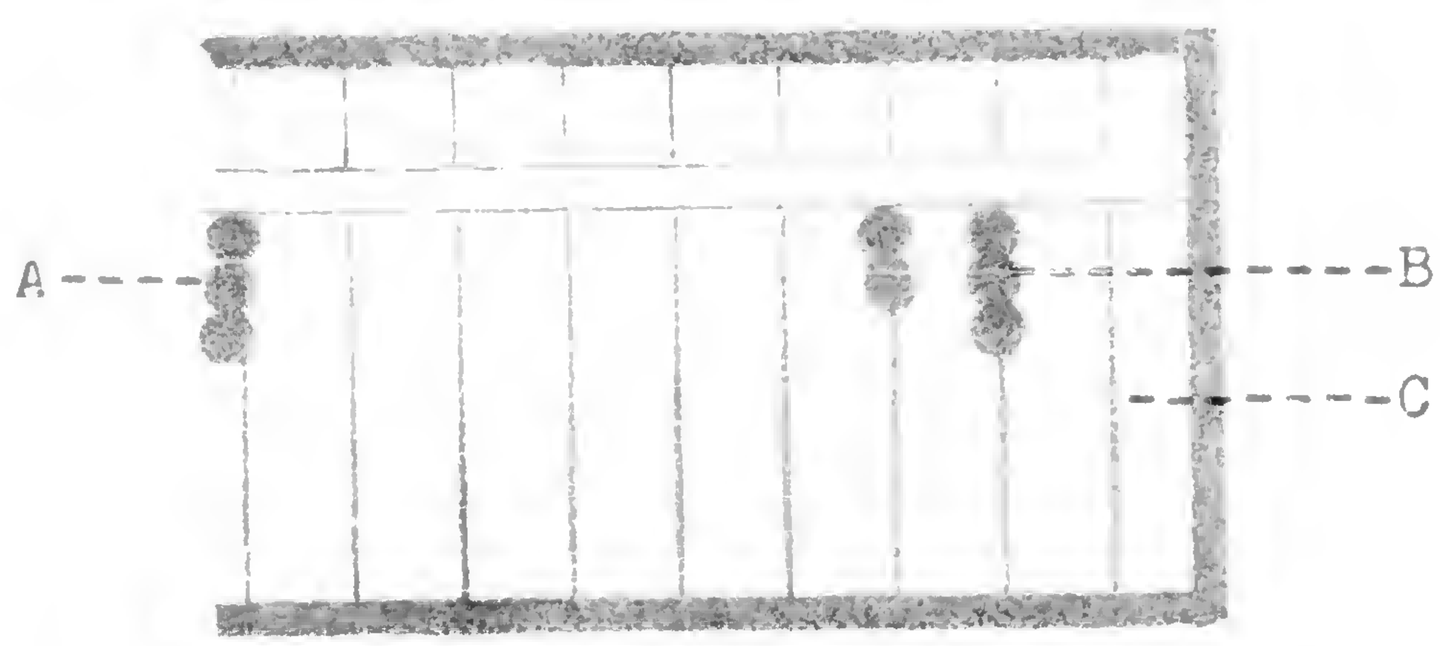
<u>Example A</u>

$$\begin{array}{r} 23 \\ \underline{\times\ 3} \\ 69 \end{array}$$

a. 3 x 3 = 9
b. 3 x 20 = 60
Product = 69

<u>On the abacus</u>:

The usual procedure is to place the multiplicand on the right-hand end of the abacus, and if it is considered necessary, the multiplier is placed to the left.



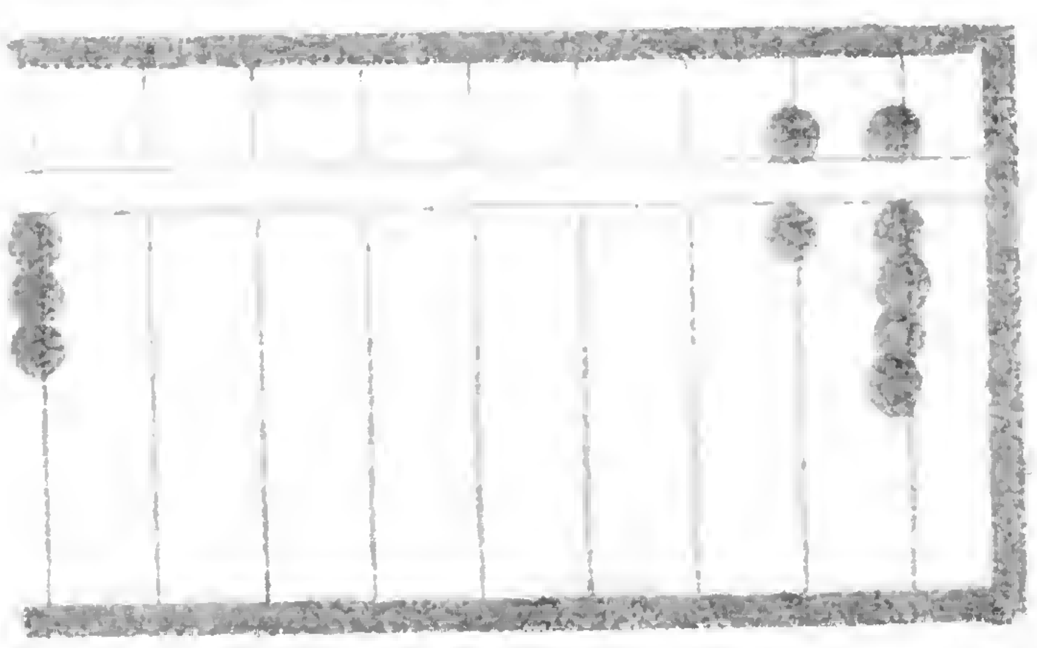
A - Multiplier
B - Multiplicand
C - Units' place of the product

When the multiplier is expressed by a one-place number the upright which is next to the units' place of the multiplicand is the units' place of the product. Multiply the multiplier by each figure of the multiplicand. The multiplicand will gradually disappear until finally only the multiplier and the product remain. If a teacher or student wishes to reverse this procedure it can be done merely by shifting the multiplicand farther to the left.

a. 3 x 3 = 9 (cancel the three of the multiplicand)

3 2 0 9

b. 20 x 3 = 60 (cancel the twenty of the multiplicand)



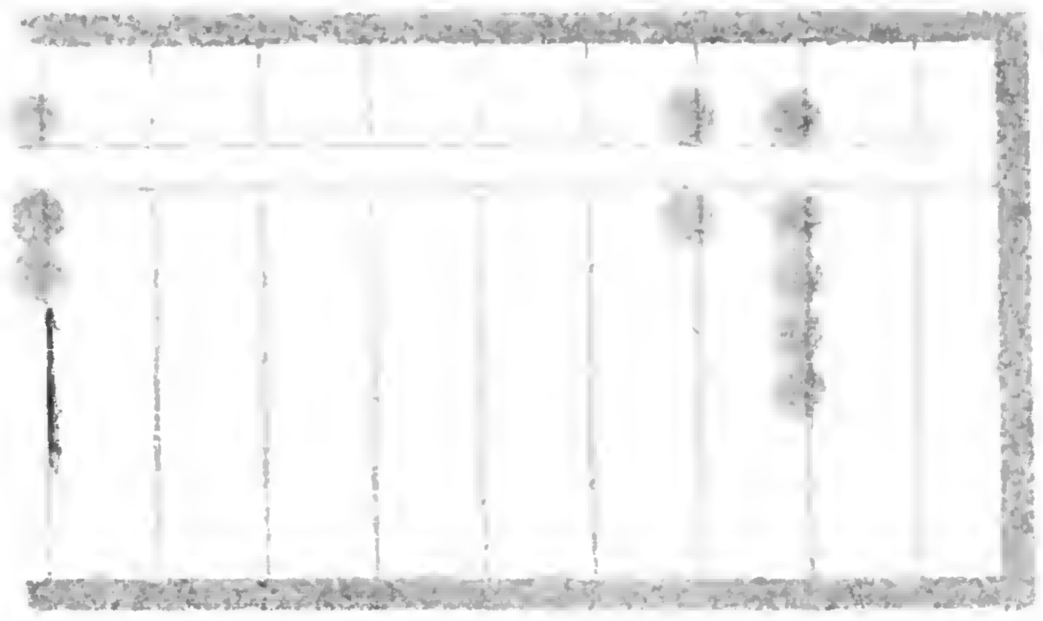
3 6 9 (the product)

Example B

$$\begin{array}{r} 69 \\ \underline{x\ 7} \\ 483 \end{array}$$

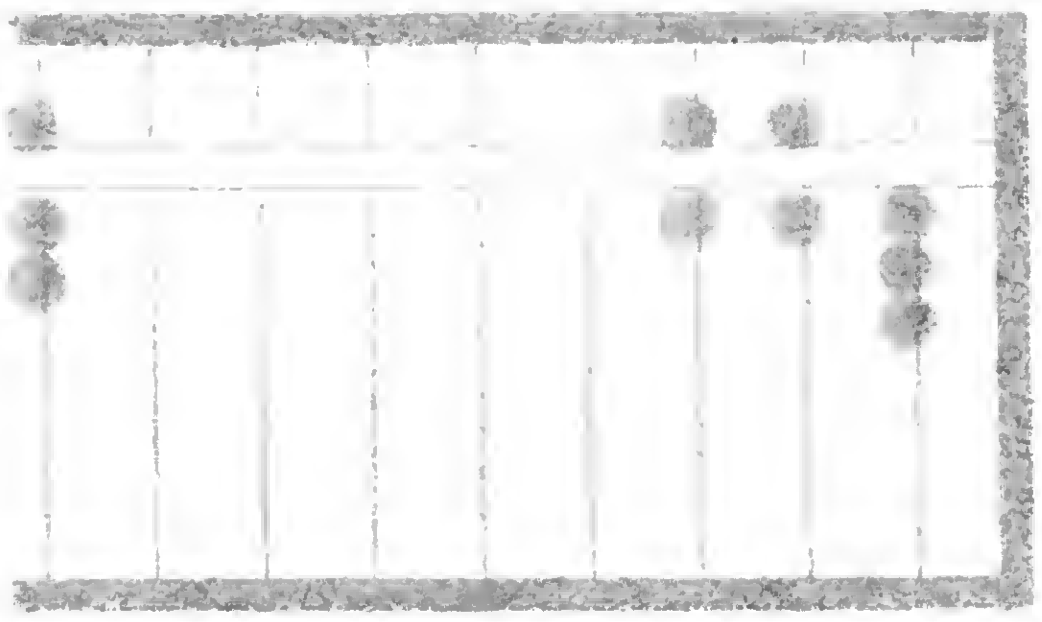
a. 7 x 9 = 63 (three and six to carry)
b. 7 x 60 = 420 + 60 (carried) = 480
Product = 483

On the abacus:



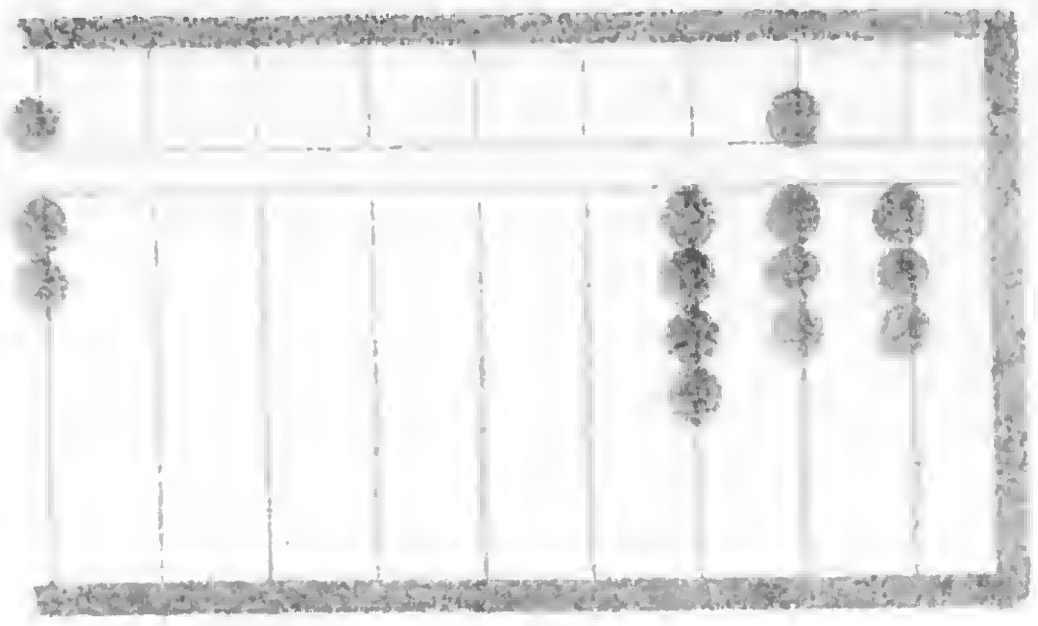
7 6 9

a. 9 x 7 (cancel the nine of the multiplicand) = 63



7 6 6 3

b. 60 x 7 (cancel the six of the multiplicand) = 420
Product = 483



7 4 8 3 (the product)

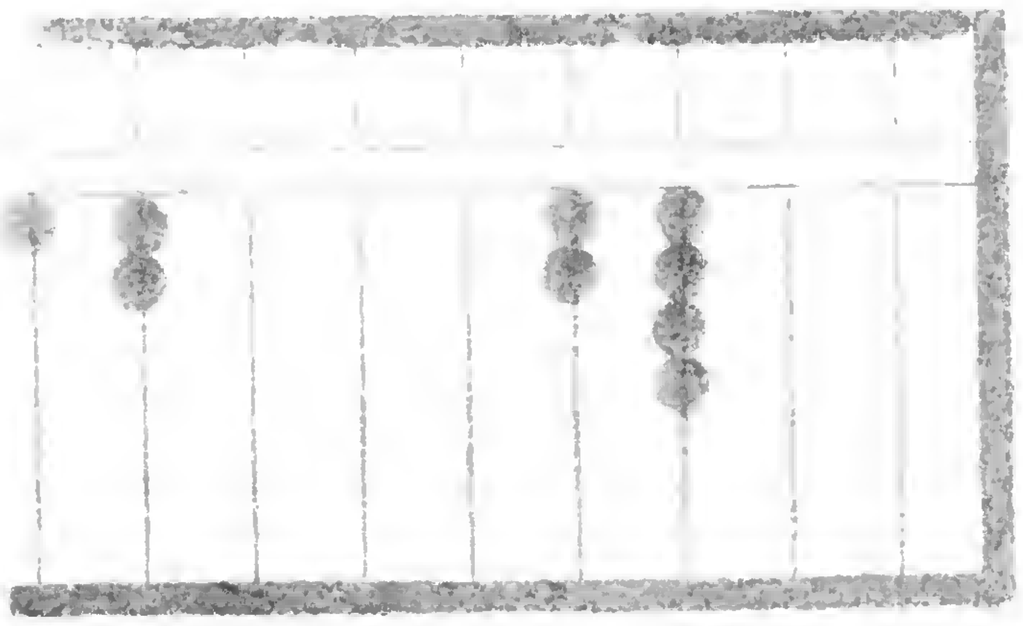
Example C

$$\begin{array}{r} 24 \\ \underline{\times\ 12} \\ 288 \end{array}$$

a. 2 x 4 = 8
b. 2 x 2 = 4
c. 1 x 4 = 4
d. 1 x 2 = 2
e. 48 + 240 = 288 (product)

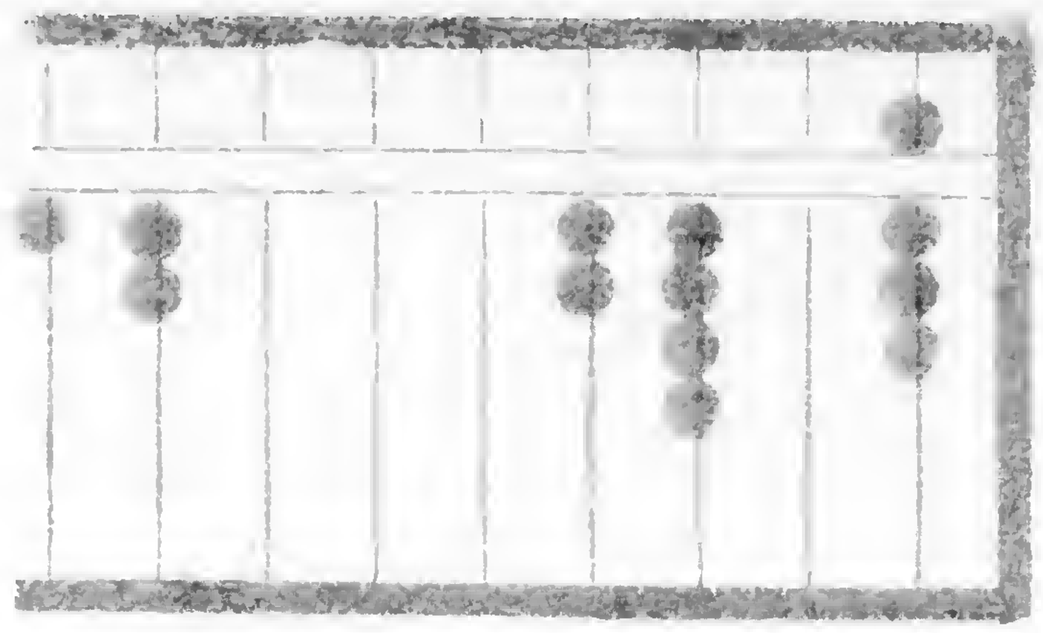
On the abacus:

When the multiplier is expressed by a two-place number the second place to the right of the units' place in the multiplicand is the units' place of the product. Multiply each figure of the multiplicand by both figures in the multiplier.



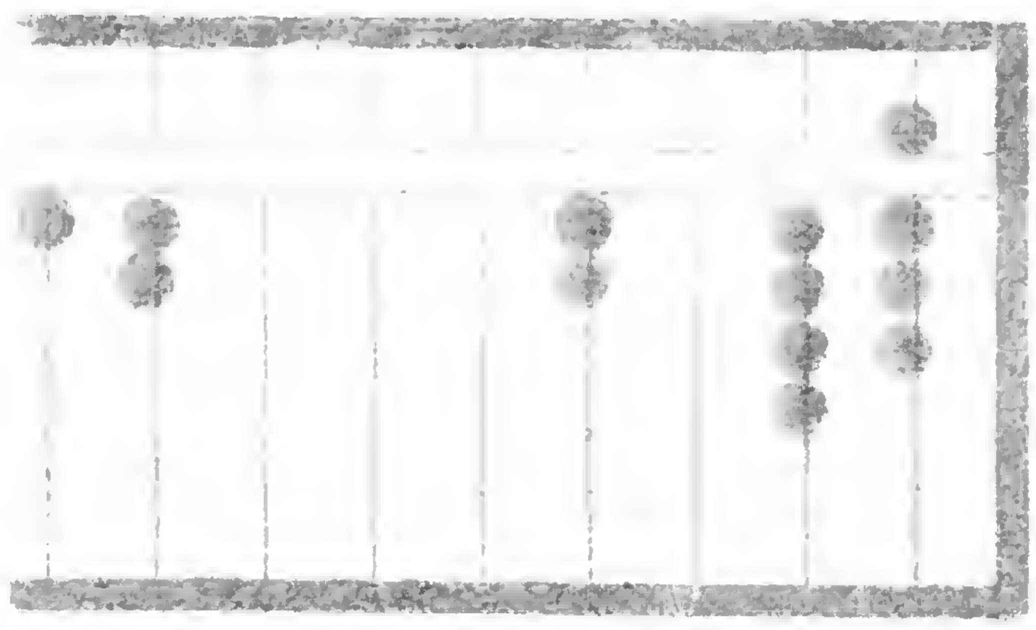
1 2 2 4

a. 4 x 2 = 8



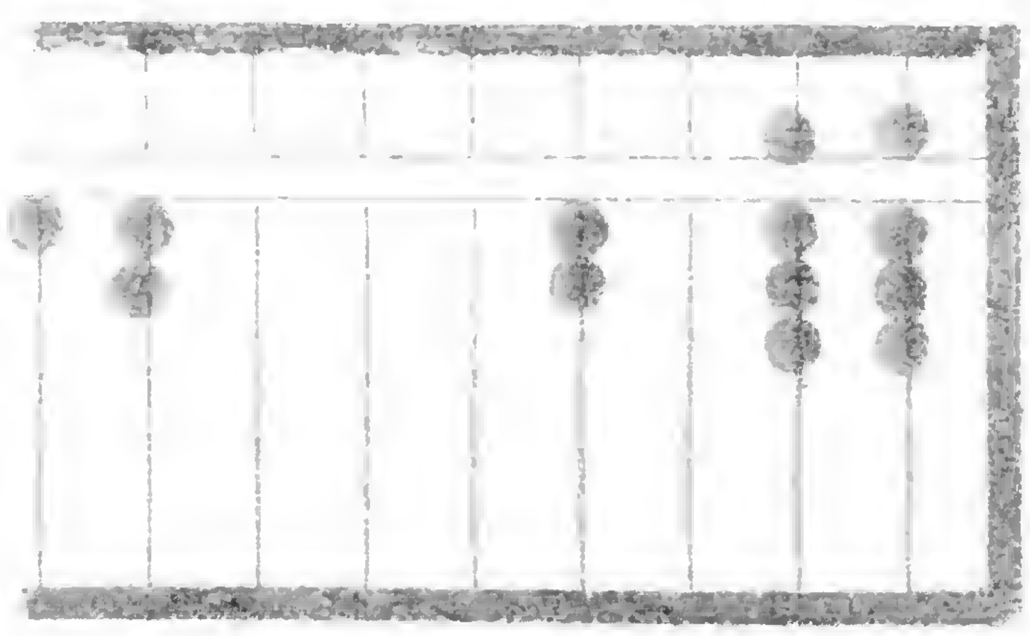
1 2 2 4 8

b. 4 x 1 = 4 (cancel the four in the multiplicand)



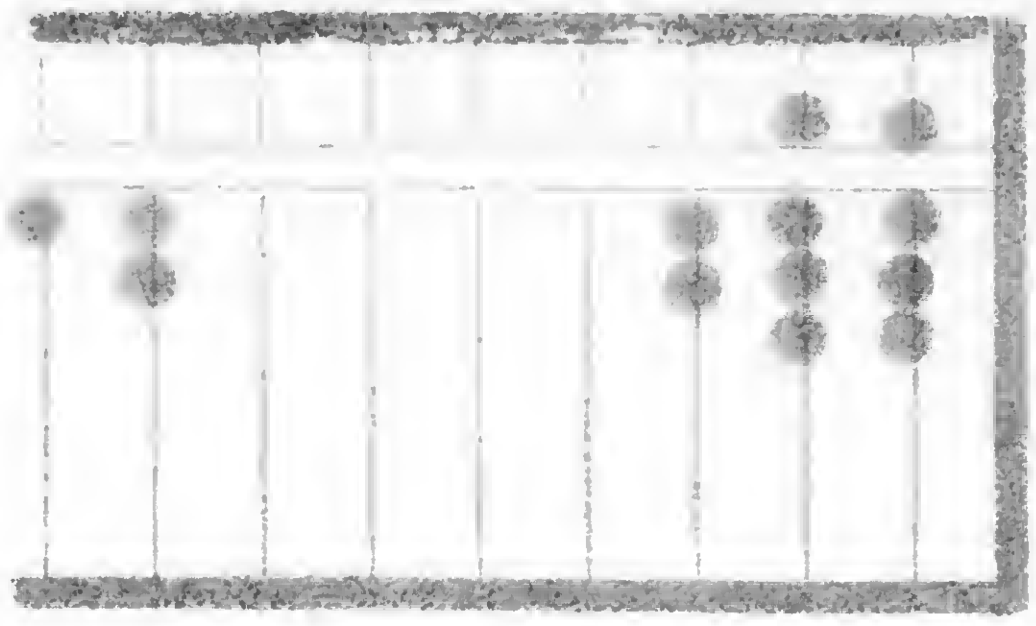
1 2 2 0 4 8

c. 2 x 2 = 4 (add the four to the number in tens' place of the product)



1 2 2 0 8 8

d. 2 x 1 = 2 (cancel the two in the multiplicand)
Product = 288



1 2 2 8 8 (the product)

The second method for multiplication is used extensively in the teaching of mental arithmetic. It begins with the first number in the multiplier and proceeds to the right.

Example A

$$\begin{array}{r} 122 \\ \times\ 6 \\ \hline 732 \end{array}$$

a. 6 x 100 = 600
b. 6 x 20 = 120
c. 6 x 2 = 12
d. 600 + 120 + 12 = 732 (the product)

On the abacus:

The primary function of the abacus in this method is to record the various partial results which must be added together in order to obtain the product. In complex problems these intermediate steps become so numerous that speed and accuracy in computation is

decreased. The abacus not only relieves the mind from the strain of remembering each step in the process, but also from the process of addition which is required to obtain the product.

600 + 120 + 12 = 732 (the product)
(See Subtraction by addition, page 75)

Example B

```
   7,398
 x     9
  66,582
```

a. 9 x 7,000 = 63,000
b. 9 x 300 = 2700
c. 9 x 90 = 810
d. 9 x 8 = 72
Therefore: 63,000 + 2700 + 810 + 72 = 66,582 (the product)
(See page 75)

On the abacus:

63,000 + 2700 + 810 + 72 = 66,582 (the product)

Example C

```
     678
    x 96
  65,088
```

a. 90 x 600 = 54,000
b. 90 x 70 = 6,300
c. 90 x 8 = 720
d. 6 x 600 = 3600
e. 6 x 70 = 420
f. 6 x 8 = 48
Therefore: 54,000 + 6300 + 720 + 3600 + 420 + 48 = 65,088 (the product)

On the abacus:

Refer to Example B.

Division

Yoshino (30) presents two methods for division in his book on the Japanese soroban. They are:

I. Division by unification

II. Division by abbreviation

Division by unification is an Oriental method which is most popular in the islands of Japan. This method makes use of both division and multiplication tables. Oriental division tables are specially designed for use with the abacus. There is nothing in Western mathematics which is directly comparable to them. Therefore, division by unification will not be discussed in this study. Its use for instructional purposes would necessitate fundamental revisions in the method which is now being taught to the blind of this country.

Division by abbreviation requires the use of multiplication tables only and is similar to the method for division which is used by both the blind and the sighted.

Example A

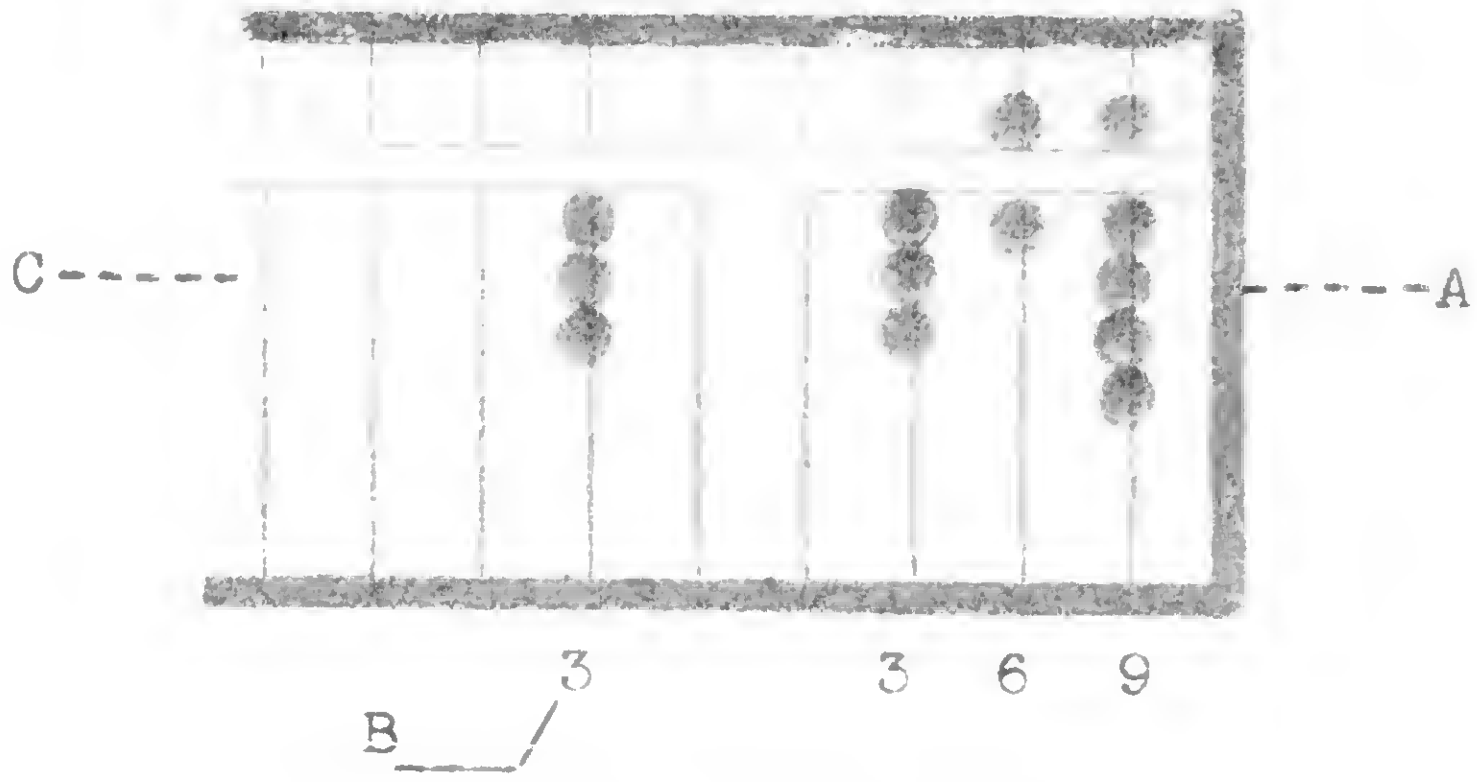
$$369 \div 3 = 123$$

a. $3 \div 3 = 1$
b. $6 \div 3 = 2$
c. $9 \div 3 = 3$
Quotient $= 123$

On the abacus:

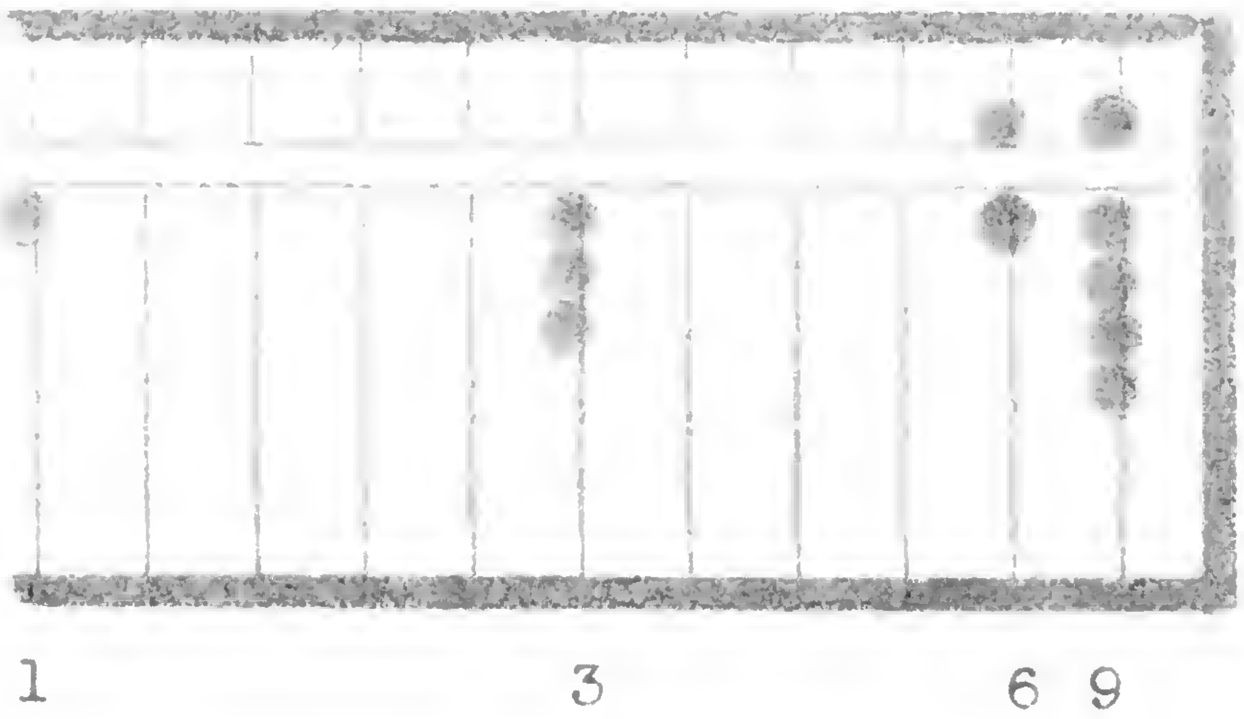
The position of the dividend on the abacus may be determined both by individual

preference and by the type of problem. In this example the dividend will be placed on the right end, the divisor in the middle, and the quotient on the left.



A - Dividend
B - Divisor
C - Quotient

a. $3 \div 3 = 1$ (cancel the three of the dividend and raise one in the quotient)

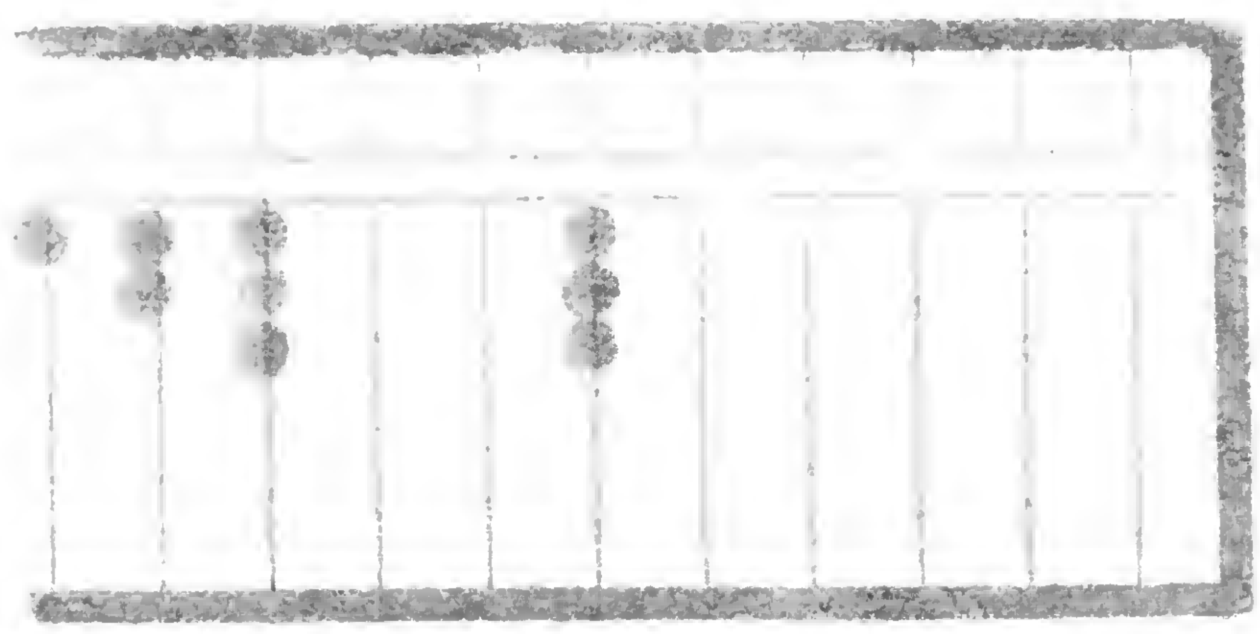


b. $6 \div 3 = 2$ (cancel the six of the dividend and raise two in the next place to the right in the quotient)

1 2 3 9

c. $9 \div 3 = 3$ (cancel the nine and place the three in the quotient to the right of the two)

Quotient = 123



1 2 3 3

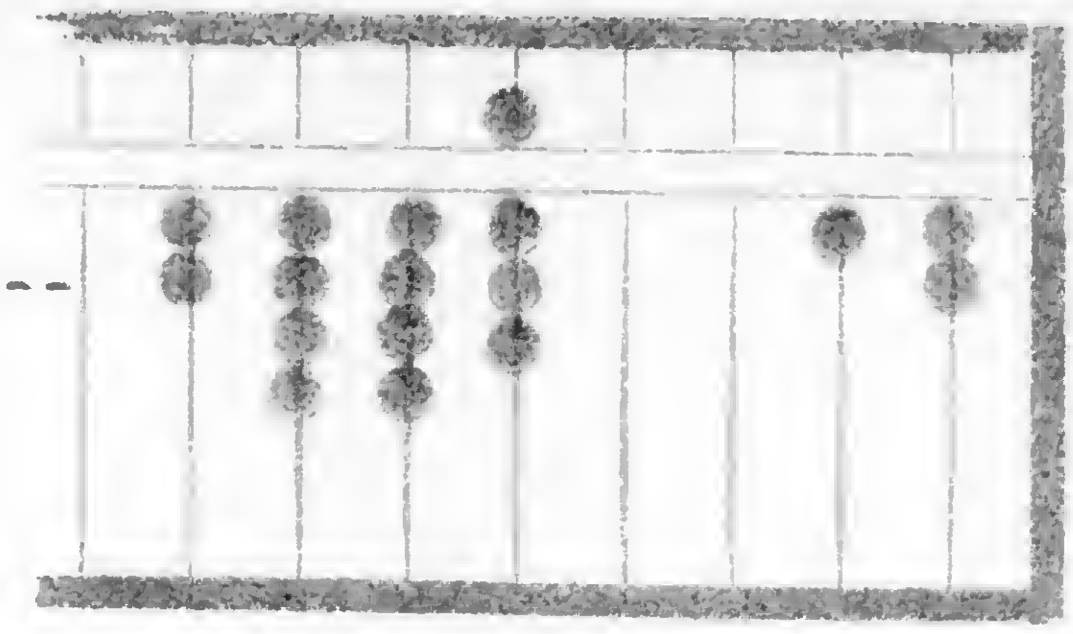
Example B

$2448 \div 12$

a. $24 \div 12 = 2$
b. $4 \div 12 = 0$
c. $48 \div 12 = 4$
Quotient = 204

On the abacus:

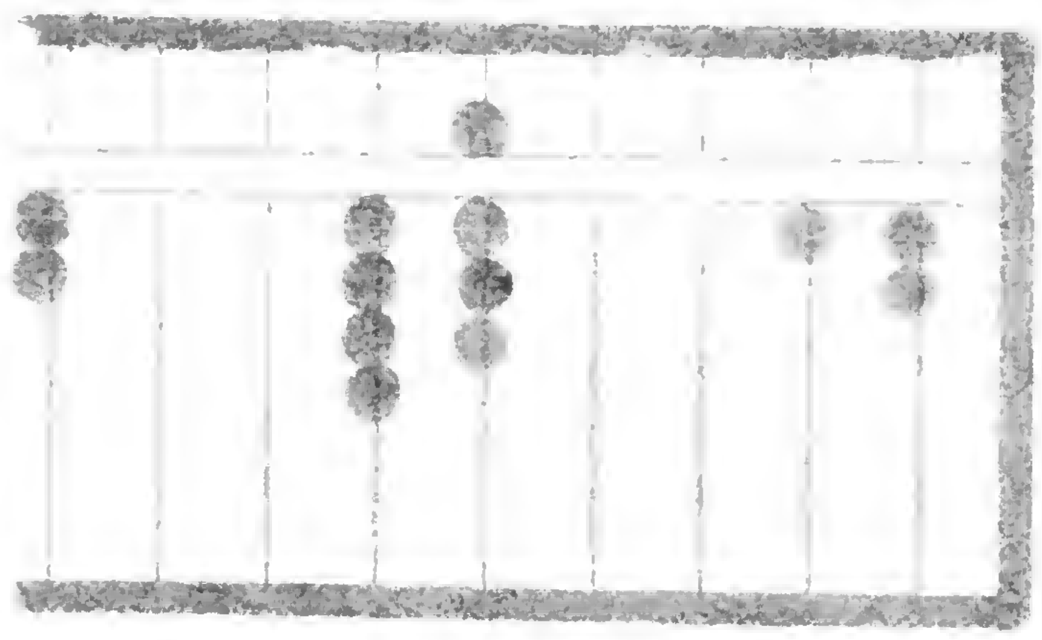
In this example the position of the dividend and the divisor are reversed.

Quotient-------  ----Divisor

2 4 4 8   1 2

Dividend_ _ _ _ _ _/

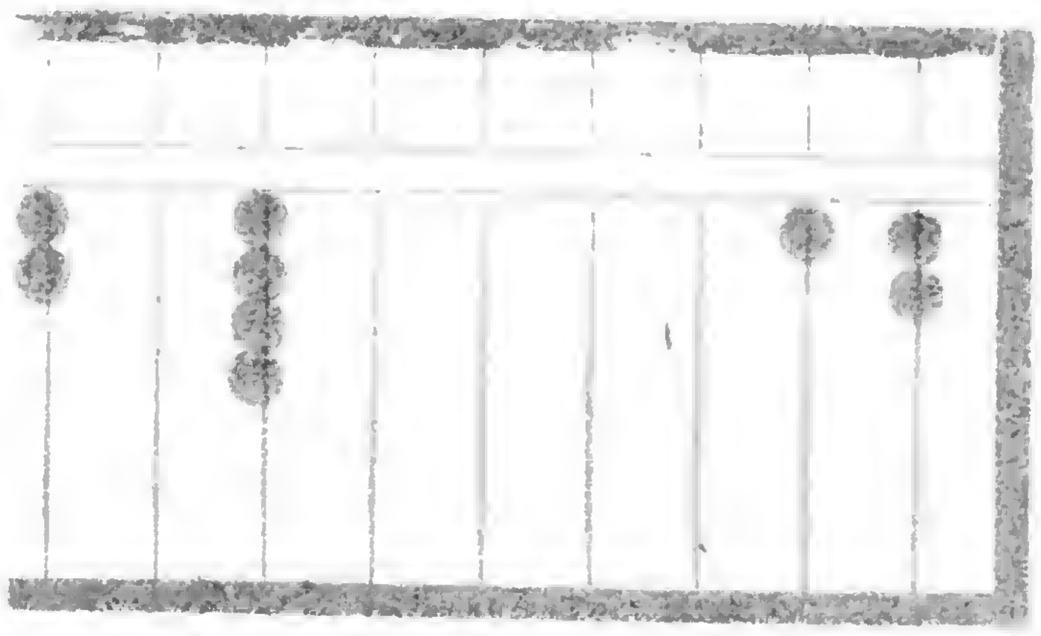
a. 24 ÷ 12 = 2 (cancel the twenty-four and raise two in the quotient)



2   4 8   1 2

b. 4 ÷ 12 = 0

c. 48 ÷ 12 = 4 (cancel the forty-eight and raise four in the quotient skipping one upright for the zero)



Quotient-----2 0 4   1 2

A fraction can be converted into a decimal number by dividing the numerator by the denominator. For example, 3/4 = .75 (three divided by four).

The fundamental operations which have been described in this chapter possess an infinite number of possible applications. The extraction of square root and cube root as well as certain processes in Algebra and Geometry may be readily performed on the abacus. The functional possibilities of this instrument need be limited only by the mathematical knowledge and ability of the person who operates it.

CONCLUSIONS

1. A relatively low level of achievement is displayed by the blind in arithmetic.
2. This poor showing appears to be the result of:
   a. unsuitable mechanical aids,
   b. disagreement concerning instructional procedures.
3. Regardless of the method employed, blind mathematicians are in need of a convenient recording device to reduce the burden of memory work which interferes with the process of computation.
4. The abacus is a mechanical aid which can be adapted to all of the various methods of computation used by the blind.
5. An adapted form of the simple bead frame possesses the greatest instructional value for number work in the elementary grades.
6. A lever adaptation of the Japanese soroban is better suited to the needs of advanced students.
7. Blind teachers and pupils consider the lever principle to be superior to that of the sliding counter.
8. The lever type abacus should be constructed in sections of nine decimal places each. A means should be provided by which one section can be supplemented

with the addition of other sections whenever necessary.

9. The functional possibilities of the abacus need be limited only by the mathematical knowledge and ability of the person who operates it.

10. The abacus would seem to be a valuable teaching aid not only for the instruction of the blind but also for use with the sighted.

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